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By JOHN D. WARMBROD
Aero-Astrodynamics Laboratory

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George C. Marshall
Space Flight Center,
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## ABSTRACT


This report presents in detail a calculation method to compute the trajectory and ablation characteristics at the stagnation point of a glass sphere entering into the atmosphere of the earth from an arbitrary point in space. The underlying equations employed by the method include the transient effects, internal radiation, melting and nonequilibrium vaporization of the glass-liquid layer. A computer program written in Fortran IV language and a detailed description of the preparation of input for this program are included. The program is particularly applicable to the study of tektites and their atmospheric entry. The method and program could easily be altered to calculate the ablation at the stagnation point of a spherical glass tip on a missile-shaped body.

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John D. Warmbrod

FLUID MECHANICS RESEARCH OFFICE
AERODYNAMICS DIVISION
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

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## DEFINITION OF SYMBOLS

(The following table lists the symbols that appear in the equations and the corresponding symbol of the computer output. The system of units is the meter-kilogram-second system where kg stands for kilogram mass.)

| Equation Symbol | Program Output Symbol | Dimensions | Description |
|---|---|---|---|
| $A_1, A_2, A_3$ $A_4$ | A1,A2,A3 A4 | | constants in vapor pressure function |
| $a_{vm}$ | | $m^2$ sec/kg | evaporation resistance |
| $a_\infty$ | | m/sec | speed of sound |
| $B_1, B_2, B_3$ $B_4$ | B1,B2,B3 B4 | | constants in viscosity function |
| $C_{w,eq.}$ | | | equilibrium mass fraction of the injected vapor at the wall |
| $c_p$ | CP | kcal/kg°K | specific heat at constant pressure of the body material |
| $C_D$ | CD | | drag coefficient, $C_D = D/(\rho_\infty W^2 A/2)$ where A is a reference area $A = \pi r_{f_o}^2$ |
| $g_\infty$ | GINF | m/sec$^2$ | gravity constant (free stream) |
| $h_e$ | HE | kcal/kg | enthalpy of air at the outer edge of boundary layer |
| $h_w$ | HW | kcal/kg | enthalpy of air at the wall |
| $h_v$ | HV | kcal/kg | heat of vaporization of the body material |
| H | H | m | geometric flight altitude |
| $H_T$ | | m | flight altitude where slip flow regime begins |

DEFINITION OF SYMBOLS (Continued)

| Equation Symbol | Program Output Symbol | Dimensions | Description |
|---|---|---|---|
| $k$ | K | kcal/m°K sec | thermal conductivity of the body material |
| $k_{air}$ | | kcal/m°K sec | thermal conductivity of air |
| $K_m$ | KM | | non-dimensional velocity gradient at outer edge of boundary layer $K_m = \frac{\frac{dU_e}{dx}(2r_{f_o})}{W}$ |
| $m$ | MASS | kg | mass of the body |
| $m_1$ | M1 | | constant in vapor pressure function |
| $m_o$ | MO | kg | initial mass of the body |
| $M$ | M | kg/kg mole | molecular weight of air (free stream) |
| $M_{vap}$ | MVAP | kg/kg mole | molecular weight of the gas vaporized |
| $M_\infty$ | MACH | | flight mach number |
| $n$ | N | | the refractive index |
| $Nu$ | | | Nusselt number |
| $P_e$ | PE | kg/m sec$^2$ | pressure of air at the outer edge of boundary layer |
| $P_\infty$ | PINF | kg/m sec$^2$ | pressure of air (free stream) |
| $P_{vap}$ | PVAP | kg/m sec$^2$ | vapor pressure of vaporizing gas |

## DEFINITION OF SYMBOLS (Continued)

| Equation Symbol | Program Output Symbol | Dimensions | Description |
|---|---|---|---|
| $P^*_{vap}$ | PVAPS | $kg/m\ sec^2$ | equilibrium vapor pressure of vaporizing gas |
| $\bar{q}_{aero}$ | QARO | $kcal/m^2\ sec$ | aerodynamic heating rate for zero vaporization |
| $q_{rad}$ | QRAD | $kcal/m^2\ sec$ | radiative heat flux rate from the body wall |
| $r_{f_o}$ | RFO | m | initial radius of the body |
| $r_f(t)$ | RFT | m | instantaneous radius of the spherical body cap |
| $R_e$ | RE | | Reynolds number based on initial body diameter |
| $R_{eff}$ | R(EFF) | | effective reflectivity of the surface |
| $t$ | TIME | sec | time |
| $\Delta t$ | DT | sec | time grid in difference method |
| $T$ | T | °K | temperature |
| $\partial T/\partial Y$ | TP | °K/m | temperature gradient in Y-direction |
| $\partial^2 T/\partial Y^2$ | TP2 | $°K/m^2$ | second derivative of temperature |
| $\partial T/\partial t$ | DTDT | °K/sec | derivative of temperature with respect to time |
| $T_\infty$ | TINF | °K | free stream temperature |
| $T_W$ | TW | °K | surface temperature of body |

DEFINITION OF SYMBOLS (Continued)

| Equation Symbol | Program Output Symbol | Dimensions | Description |
|---|---|---|---|
| $T_o$ | TO | °K | temperature of body at initial time |
| $T_e$ | TE | °K | temperature of air at outer edge of boundary layer |
| U | UINF | m/sec | horizontal free stream velocity component |
| dU/dt | DU/Dt | $m/sec^2$ | derivative of flight velocity component with respect to time |
| V | VINF | m/sec | vertical free stream velocity component |
| dV/dt | DV/Dt | $m/sec^2$ | derivative of flight velocity component with respect to time |
| v | V | m/sec | ablation rate |
| $v_w$ | | m/sec | vaporization rate $v_w = v(Y=0)$ |
| $v_\infty$ | | m/sec | total ablation rate at each time $v_\infty = v(Y=Y_o)$ |
| W | WINF | m/sec | free stream flight speed |
| dW/dt | DW/Dt | $m/sec^2$ | acceleration |
| $W_c$ | | m/sec | circular speed of the earth |
| Y | Y | m | coordinate measured from body surface |
| $\Delta Y$ | DY | m | distance grid in difference method |

DEFINITION OF SYMBOLS (Continued)

| Equation Symbol | Program Output Symbol | Dimensions | Description |
| --- | --- | --- | --- |
| $Y_B$ | YB | m | upper limit of integral in heat balance equation |
| $Y_o$ | | m | point in the body where no melting occurs |
| $Y_s$ | YS | m | thickness of body material lost due to ablation |
| $Y_{s_w}$ | YSW | m | thickness of body material lost due to vaporization |
| $\alpha$ | ALF | 1/m | reciprocal radiation mean free path |
| $\alpha_A$ | ALFA | 1/m | absorption coefficient |
| $\alpha_v$ | ALFV | | vaporization coefficient |
| $\epsilon$ | E | | emissivity constant of the opaque body material |
| $\phi$ | PHI | degrees | angle of attack measured from horizontal |
| $\mu$ | MU | kg/m sec | viscosity of the body material |
| $\mu_e$ | MUE | kg/m sec | viscosity of air at outer edge of boundary layer |
| $\mu_\infty$ | MUINF | kg/m sec | viscosity of air (free stream) |
| $\rho$ | RHO | $kg/m^3$ | density of the body material |
| $\rho_\infty$ | RINF | $kg/m^3$ | density of air (free stream) |
| $\rho_e$ | RHOE | $kg/m^3$ | density of air at the outer edge of boundary layer |

DEFINITION OF SYMBOLS (Continued)

| Equation Symbol | Program Output Symbol | Dimensions | Description |
|---|---|---|---|
| $\tau_w/X$ | TAW | $kg/m^2\ sec^3$ | shearing stress at wall divided by X |
| $\psi$ | SI | | heat blockage factor, a correlation function derived from solutions to the boundary layer equations |
| | INT | °K m/sec | value of integral in heat balance equation |
| $\beta_1$ | BETA1 | | constant in shear stress relation |
| $\partial F/\partial Y$ | DF | | see equation (A-11) |

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# A CALCULATION METHOD FOR THE ABLATION OF GLASS-TIPPED BLUNT BODIES


## SUMMARY

This report presents in detail a calculation method to compute the trajectory and ablation characteristics at the stagnation point of a glass sphere entering into the atmosphere of the earth from an arbitrary point in space. The underlying equations employed by the method include the transient effects, internal radiation, melting and nonequilibrium vaporization of the glass-liquid layer. A computer program written in Fortran IV language and a detailed description of the preparation of input for this program are included. The program is particularly applicable to the study of tektites and their atmospheric entry. The method and program could easily be altered to calculate the ablation at the stagnation point of a spherical glass tip on a missile-shaped body.


## I. INTRODUCTION

A great many investigations have been conducted in the past eight years to determine the ablation of bodies entering the earth's atmosphere at hypersonic speeds. For glassy or quartz-like material, the ablation follows a process of both melting and vaporization. Sutton [15] in 1958 obtained the first exact numerical solution for the steady state ablation of glass-like material at the stagnation point of a body subjected to hypersonic flight conditions. His method is limited, however, in that it accounts for melting only, and even for this restricted case, it is very difficult to employ. Bethe and Adams [2], Few and Fanucci [10], Scala [14], and many others presented quasi-steady solutions that accounted for both melting and vaporization. Adams [1] presented a calculation method that employed a finite difference procedure. This procedure accounted for the transient effects of the ablation problem which included both melting and vaporization effects. Scala and Vidale [16], who considered the nonequilibrium effects of vaporization in the quasi-steady approximation found that, when a material is subjected to severe heating conditions, the net rate of ablation due to vaporization may be diffusion controlled, kinematically limited, or both. Kadanoff [9] developed a method for calculating the net radiative flux within a semi-infinite body which emits, absorbs, and scatters this radiation and which allows some radiation to escape from its surface. Chapman [3] in 1963 presented a condensed account of a nonsteady calculation method which

accounts for melting, vaporization, and internal transport of radiant energy. Chapman includes approximately the effects of the presence of $O_2$ and hence production of the $S_iO$ vapor as given by Hidalgo [7].

A finite-difference method is presented for the solution of the ablation problem in the vicinity of the stagnation point which accounts for transient effects, nonequilibrium vaporization, and internal radiation for a spherically or hemispherically shaped body composed of a glassy or quartz-like material that melts and vaporizes. The equations developed by Chapman [4] for entry into a planetary atmosphere are employed in this calculation scheme to compute the trajectory of the body entering the earth's atmosphere. The equations given in the following sections of this paper are a system that must be calculated for each time step and are presented in the order of computation in the scheme. At each time step, there is an iteration involving $T_W$ (the surface temperature) such that a heat balance equation at the surface is satisfied.

A computer program written in Fortran IV language for the equations in this report is presented in the appendix. This program is designed to calculate the trajectory, temperatures, and ablation history at the stagnation point of a sphere or hemisphere composed of a glassy material which is entering into the atmosphere of the earth. The preparation of 10 input data cards is required for each particular example to be computed. The cost of running this program on the 7094 computer at Marshall Space Flight Center is approximately one cent per time step for the case without internal radiation and 2.5 cents per time step for the case with internal radiation.

The author is indebted to Mr. Verkuel Eubanks of the Computation Laboratory, General Electric Company, Huntsville, Alabama for the detailed programming and valuable suggestions concerning the numerical solution of this problem.

## II. INITIAL CONDITIONS AND CALCULATION PROCEDURE

The entry into the earth's atmosphere begins at an initial altitude $H_o$ (an input value, usually 150,000 m); i.e., t = o when $H = H_o$. Other initial conditions which must be given are angle of attack, $\phi_o$, initial flight velocity, $W_o$, radius of the sphere or hemisphere, $r_{f_o}$, and the physical properties of the body. The preparation of these input data for the computer program is outlined in Appendix B. At the initial time it is assumed that the body has a uniform temperature $T_o$ = 300°K throughout the body.

The differential equations describing the flow of the viscous glass layer in the vicinity of the stagnation point are given in Appendix A of this paper. The mathematical procedure solves each of these three basic conservation equations by taking small step-by-step increments in time. The equations presented in the following sections are a system that must be calculated for each time step; it is assumed that the calculations are being carried out at time t and that the solution is known at time t - $\Delta t$. In order for convergence of the forward difference procedure in the solution of the transient energy equation, the following relationship between grid and material properties must be satisfied:

$$\frac{\Delta t}{(\Delta Y)^2} \leq \frac{\rho c_p}{\pi k}$$

where the material properties $\rho$, $c_p$, and k are assumed constant and not a function of temperature. If the above relationship is not satisfied, the program will solve the equation with the equality sign for $\Delta Y$, and the calculation will continue.

The computer program contains an option of whether to account for or disregard the effects of internal radiation. The first page of the computer output is printed for the appropriate case as:

(a) THE ABLATION PROGRAM WITH INTERNAL RADIATION

(b) THE ABLATION PROGRAM WITHOUT INTERNAL RADIATION.

The case without internal radiation assumes the body to be composed of an opaque material, and only heat radiated away from the surface is accounted for. The case with internal radiation uses the equations derived by Kadanoff [9].

Calculation by the computer program ends when one of the following conditions are met:

(1) The body reaches the surface of the earth ($H \leq o$).

(2) The body is dissipated due to ablation.

(3) The altitude H exceeds a predetermined maximum. This is to avoid a body from bouncing out of the earth's atmosphere and continuing indefinitely.

(4) The flight time reaches a predetermined limit.

(5) An error condition exists.

In each case an appropriate message is printed explaining the reason for terminating the calculation.

The program allows for up to three different time steps and print frequencies during any one run. For instance, let $\Delta t$ be the delta time, $T_m$ be the upper time limit at that time step, and $M_p$ be an integer denoting the number of time steps between prints.

| Time Step | Maximum Time | Print Frequency |
|---|---|---|
| $\Delta t_1$ | $T_{m_1}$ | $M_{p_1}$ |
| $\Delta t_2$ | $T_{m_2}$ | $M_{p_2}$ |
| $\Delta t_3$ | $T_{m_3}$ | $M_{p_3}$ |

The program would use a $\Delta t$ of $\Delta t_1$ between time = 0 and time = $T_{m_1}$, printing every $M_{p_1}$ steps. The program would then use $\Delta t_2$ between $T_{m_1}$ and $T_{m_2}$, etc. The program selects the largest of the $T_m$ values as the maximum time for the entire run and selects the largest $\Delta t$ values to check the grid ratio discussed above. If one wants to hold the print interval and $\Delta t$ constant throughout a run, it is best to set $T_{m_1}$ equal to end of flight time and $T_{m_2}$ and $T_{m_3}$ equal to 0.

The subsequent sections, which give the equations used in the calculation method, are presented in the order that the computer program actually solves the problem. Since most of the underlying analysis of the equations is well known, they are presented in their final form. Many of the relations were taken from other references dealing with aerodynamics, ablation, and boundary layer theory.

The computed results by this program for a typical atmospheric entry of an initially spherical-shaped tektite are shown in Figure 1. The initial flight conditions and physical properties for this example are

(1) case with internal radiation

(2) $H_o$ = 150,000 m

(3) $W_o$ = 11,200 m/sec

(4) $\phi_o$ = -20°

(5) $r_{f_o} = .01$ m

(6) $k = 3.75 \times 10^{-4}$ kcal/(m °K sec)

(7) $\rho = 2400$ kg/m$^3$

(8) $c_p = .34$ kcal/(kg °K)

(9) $h_v = 3050$ kcal/kg

(10) $M_{vap} = 40.278$ kg/kg-mole

(11) constants in viscosity function (see equation (78))

$B_1 = .1$

$B_2 = 27,620$

$B_3 = 262$

$B_4 = -9.09$

(12) constants in equilibrium vapor pressure function (see equation (80))

$A_1 = 101,325$

$A_2 = 0$

$A_3 = -57,800$

$A_4 = 19.1$

(13) $\beta_1 = .28$

(14) $\alpha_v = \infty$

(15) $\alpha = 1900$ 1/m

(16) $\alpha_A = 285$ 1/m

(17) $n = 1.5$

(18) $R_{eff} = .2$

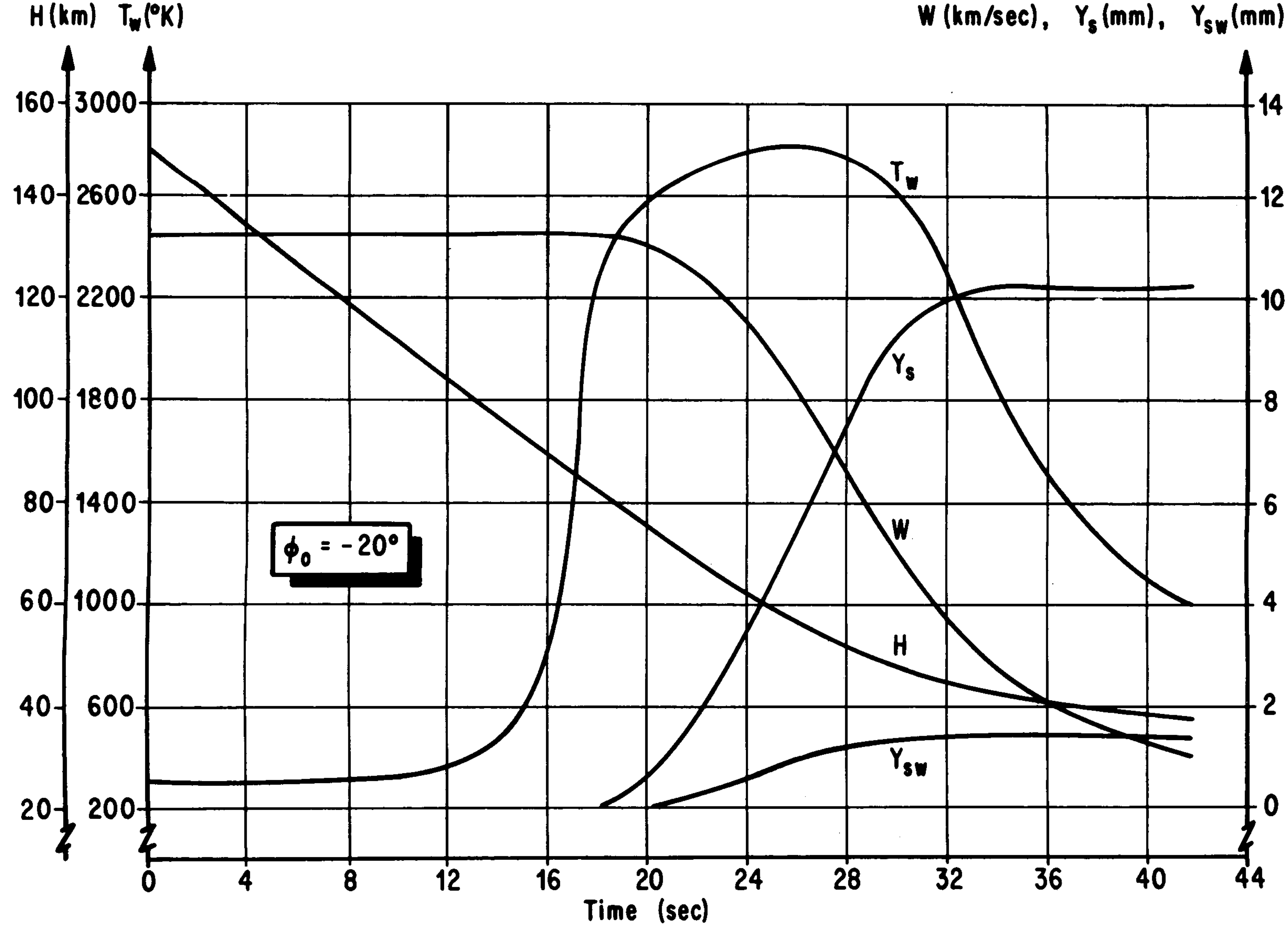



FIG. 1. COMPUTED TRAJECTORY AND ABLATION RESULTS FOR A SPHERE OF 10 MM RADIUS DURING ENTRY INTO THE EARTH'S ATMOSPHERE

## III. RADIUS OF CURVATURE OF THE FRONT FACE OF THE BODY

At the initial time, the body is assumed to be either a sphere or hemisphere. During the entry of the body into the atmosphere, the radius of curvature $r_f(t)$ varies with the extent of ablation $Y_s(t)$. For bodies where the ablated thickness $Y_s(t)$ is small compared to the initial radius of curvature $r_f(t)$ such as the ablation of the front face of a blunt missile body, the variation of $r_f(t)$ can be neglected in the calculation scheme. For small bodies such as tektites where the ablation thickness is comparable to the body radius, the variation of $r_f(t)$ is important and therefore must be determined at each time step. Two methods of determining this radius of curvature are given below. One method (Method 1) gives results which overestimate the total mass lost and the other method (Method 2) probably underestimates the total mass lost. The program is coded such that either method can be used in the calculation scheme.

### A. Method 1

This method assumes that the mass lost by the body is the mass lost due to both melting and vaporization; i.e., the material that is melted and vaporized is removed from the body. The assumed body shape is shown in the following illustration (Figure 2).

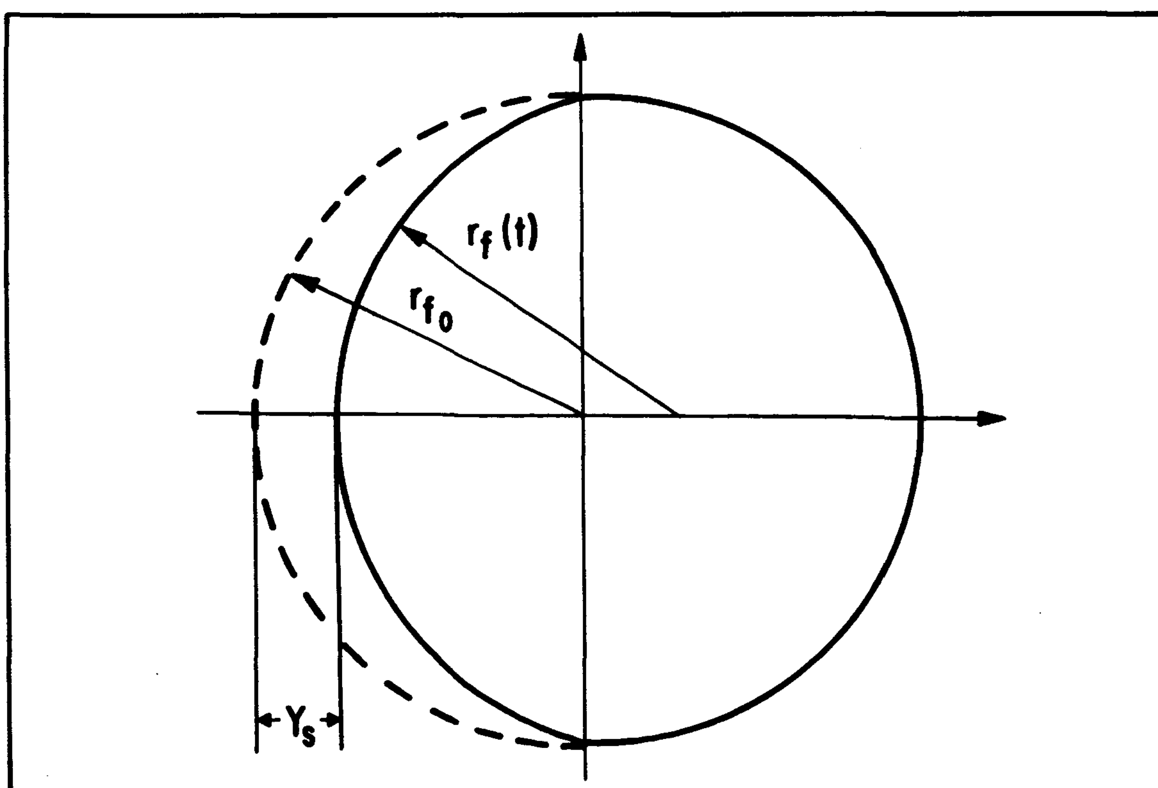



Figure 2. The Body Shape for Method 1

The radius of curvature is given by

$$r_f(t) = r_{f_o} + \frac{1}{2}\frac{Y_s^2}{r_{f_o} - Y_s}, \tag{1}$$

where the total thickness along the axis of symmetry that is lost up to the time t due to the ablation is

$$Y_s(t) = -\int_o^t v_\infty \, dt \tag{2}$$

and $v_\infty(t)$ is the instantaneous ablation velocity. When the condition $Y_s(t) \geqq r_{f_o}$ is met, the computer program is coded such that the calculation stops and a message is printed out denoting that half (all) of the sphere (hemisphere) has been dissipated due to ablation.

B. Method 2

Chapman [3] in his experimental studies on the ablation of a tektite glass sphere placed in a hypervelocity arc jet, found that the melting begins at the stagnation point where the heating is most severe and the molten material flowed around the sphere and solidified on top of the original spherical surfaces. The mathematical model for this body shape is shown in Figure 3. It is assumed that the mass loss of the body is the mass lost due to vaporization only and that the mass melted is equal to the mass of the flanges. Chapman derived the following empirical relation for the radius of curvature $r_f(t)$ as a function of ablated thickness $Y_s(t)$ from the experimental results:

$$r_f(t) = r_{f_o}\left\{1 + .5\left[1 - \exp\left(-\frac{10Y_s(t)}{r_{f_o}}\right)\right] - .32\left(\frac{Y_s(t)}{r_{f_o}}\right)^2\right\}. \tag{3}$$

The thickness of body material that has been lost due to vaporization is

$$Y_{s_w} = -\int_o^t v_w \, dt, \tag{4}$$

where $v_w(t)$ is the instantaneous vaporization rate at the surface of the body at the stagnation point.

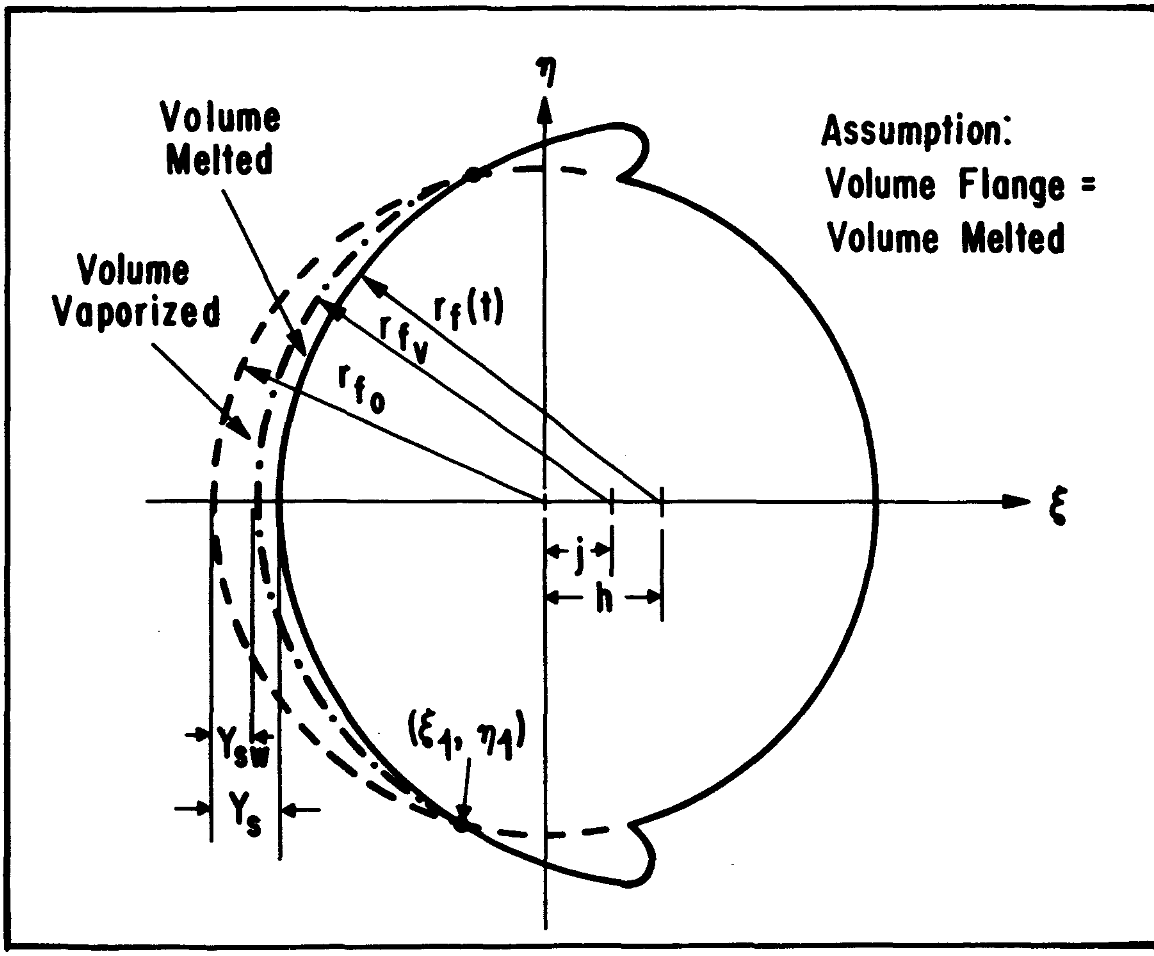



Figure 3. The Body Shape for Method 2

C. <u>Calculation of the Body Thickness</u>

The thickness of the body S(t) measured along the axis of symmetry is given at each time by

$$S(t) = L(t) \cdot \Delta Y.$$

The initial thickness of the body is given by $S_o = L_o \cdot \Delta Y$.

$$L_o = \frac{i \cdot r_{f_o}}{\Delta Y} ,$$

where i = 1 for a hemisphere and i = 2 for a sphere. The computer program approximates the function L(t) by taking it as the integer part of the number

$$\frac{S_o - Y_s(t)}{\Delta Y} + .5.$$

Therefore, L(t) + 1 is the number of points Y along the axis of symmetry that are considered in the calculation at time t.

## IV. THE TRAJECTORY OF THE ENTRY BODY

By neglecting the lateral forces, Chapman [4] studied the descent of a body in a meridian plane of a spherically symmetric atmosphere about a spherically symmetric planet. The coordinate system and velocity components are shown in Figure 4.

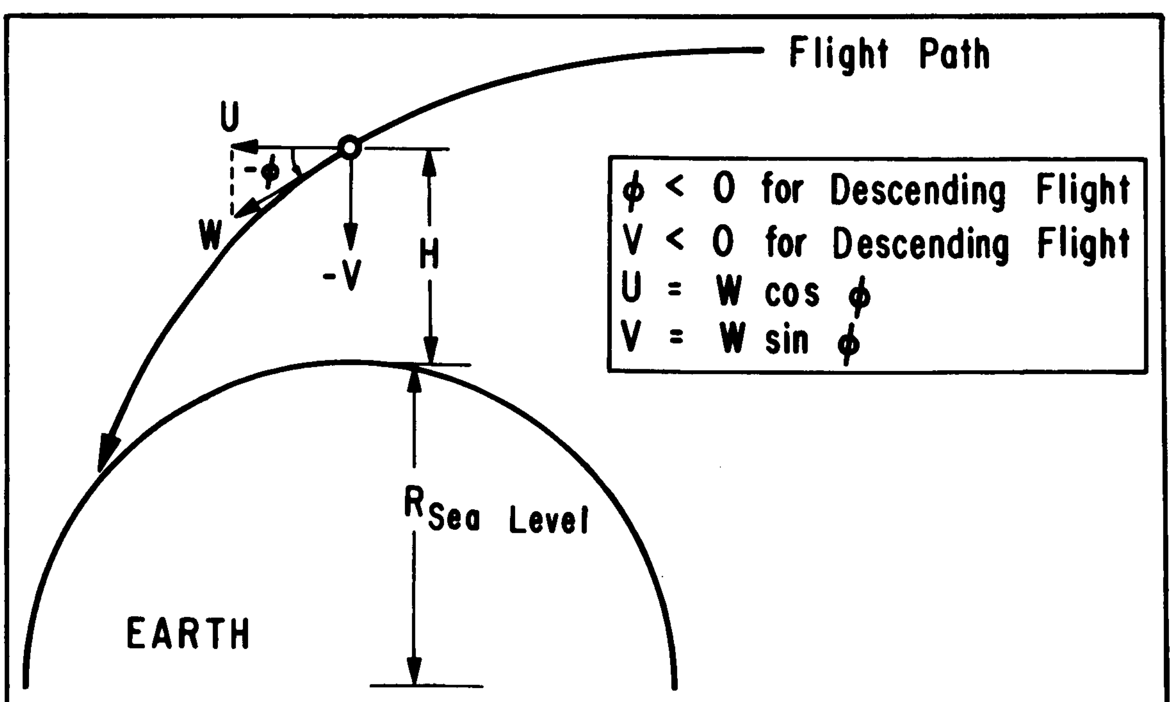



Figure 4. Coordinate System for Trajectory Calculations

For a spherical body the resulting equations of motion are as follows:

$$\frac{dU(t)}{dt} = \frac{-U(t)\ V(t)}{H(t) + R_{sea\ level}} - \frac{1}{m(t)} \left[ \frac{\rho_\infty(t)}{2} C_D(t) \left( U^2(t) + V^2(t) \right) + .75\ \rho v_w^2(t) \right] \frac{U(t)\ \pi r_{f_o}^2}{\sqrt{U^2(t) + V^2(t)}} \tag{5a}$$

$$\frac{dV(t)}{dt} = -\ g_\infty(t) + \frac{U^2(t)}{H(t) + R_{sea\ level}} - \left[ \frac{\rho_\infty(t)}{2} C_D(t) \left( U^2(t) + V^2(t) \right) + .75\ \rho v_w^2(t) \right] \frac{V(t)\ \pi r_{f_o}^2}{m(t) \sqrt{U^2(t) + V^2(t)}}, \tag{5b}$$

where

(a) $H(t) = H(t - \Delta t) + V(t) \cdot \Delta t$ is the local geometric altitude.

(b) $g_\infty(t) = g_{sea\ level} \left( \frac{H(t) + R_{sea\ level}}{R_{sea\ level}} \right)^{-2}$ is the local value of gravitational acceleration. $R_{sea\ level} = 6{,}380{,}000$ m and $g_{sea\ level} = 9.80665\ m/sec^2$ for the system of units used herein.

(c) $\rho_\infty(t)$ = the density of air in free stream, a function of $H(t)$.

(d) $\rho$ = the density of the body material

(e) $C_D(t)$ = the drag coefficient.

The ordinary differential equations (5a) and (5b) are numerically solved by the method of Runge-Kutta at each time t for the flight velocity components U(t) and V(t). To avoid including the above differential equations in the iteration procedure for the surface temperature $T_w(t)$, it is assumed that, in (5a) and (5b), $v_w(t) = v_w(t - \Delta t)$ and $m(t) = m(t - \Delta t)$ which is a good approximation due to the smallness of the time step $\Delta t$.

Relations for the drag coefficient $C_D(t)$ for a sphere and a hemisphere were derived by empirically fitting curves from various sources. These formulas for different flight conditions and regimes (see Section VIB) are as follows:

(a) Molecular Flow Regime ($H > H_T$) for Spheres and Hemispheres

(1) $0 \leqq M_\infty \leqq 9$; $C_D = \frac{2.85}{M_\infty} + 1.68$

(2) $9 < M_\infty \leqq \infty$; $C_D = 2.0$.

(b) Continuum Flow Regime ($H < H_T$) for Spheres

(1) $0 \leqq M_\infty \leqq .8$; $C_D = .5$

(2) $.8 < M_\infty \leqq 1.26$; $C_D = .812\, M_\infty - .023$

(3) $1.26 < M_\infty \leqq 2.0$; $C_D = 1.034 - .027\, M_\infty$

(4) $2.0 < M_\infty \leqq \infty$; $C_D = .9 + M_\infty/\sqrt{Re}$ .

(c) Continuum Flow Regime ($H < H_T$) for Hemispheres

(1) $0 \leqq M_\infty \leqq 2.0$; $C_D = 1.35$

(2) $2 < M_\infty \leqq \infty$; $C_D = 1.35 + M_\infty/\sqrt{Re}$ .

The free stream Mach number and Reynolds number are given by

$$M_\infty(t) = \frac{W(t)}{a_\infty(t)} \tag{6}$$

$$R_e(t) = \frac{\rho_\infty(t)\ W(t)\ 2r_{f_o}}{\mu_\infty(t)} \tag{7}$$

where the atmosphere properties $\rho_\infty$, $a_\infty$, $T_\infty$, and $\mu_\infty$ are given as a function of the flight altitude, H. The computer program employs a subroutine that contains a table of atmospheric properties taken from Reference 17. This subroutine interpolates the table for the properties at the given value of H for each time. After solution of the two differential equations (5a) and (5b), the following relations are calculated:

(a) The resultant velocity

$$W(t) = \sqrt{U^2(t) + V^2(t)}\ . \tag{8}$$

(b) The angle of attack (see Figure 4)

$$\emptyset(t) = \tan^{-1}\left[V(t)/U(t)\right]. \tag{9}$$

(c) The acceleration of the body

$$\frac{\partial W}{\partial t} = \frac{U\dfrac{\partial U}{\partial t} + V\dfrac{\partial V}{\partial t}}{W}\ . \tag{10}$$

## V. PROPERTIES OF AIR BEHIND THE NORMAL SHOCK

Stagnation point properties of air behind the normal shock, or at the outer edge of the air boundary layer, that are needed in the ablation problem are approximated by curve fits of curves and tabulated data for air in thermal and chemical equilibrium published in Reference 8. The properties and their respective curve fits are as follows:

### A. Temperature

(1) For $W > 2100$ (m/sec) and $M_\infty < 35$,

$$\frac{T_e}{T_\infty} = \frac{T_e}{T_{e_{ideal}}} \frac{T_{e_{ideal}}}{T_\infty}, \tag{11}$$

where

$$\frac{T_{e_{ideal}}}{T_\infty} = 1 + \frac{M_\infty^2}{5}. \tag{12}$$

The ratio of real to ideal gas effects is given by

$$\frac{T_e}{T_{e_{ideal}}} = d_o + d_1 M_\infty + d_2 M_\infty^2 + d_3 M_\infty^3 + d_4 M_\infty^4 + d_5 M_\infty^5, \tag{13}$$

where

$$d_o = 4.016949 - 1.49287 \times 10^{-5}\ H$$

$$d_1 = -.895475 + 4.51127 \times 10^{-6}\ H$$

$$d_2 = 9.28796 \times 10^{-2} - .54521 \times 10^{-6}\ H$$

$$d_3 = -4.746323 \times 10^{-3} + 3.05088 \times 10^{-8}\ H$$

$$d_4 = 11.6111955 \times 10^{-5} - 7.96241 \times 10^{-10}\ H$$

$$d_5 = -10.86105 \times 10^{-7} + 7.84415 \times 10^{-12}\ H$$

and $H = H(m)$ is the flight altitude.

(2) For $W \leqq 2100$ (m/sec) and $M_\infty < 35$,

$$\frac{T_e}{T_{e_{ideal}}} = 1. \tag{14}$$

(3) For $M_\infty \geqq 35$,

$$\frac{T_e}{T_\infty} = 45. \tag{15}$$

B. Density

(1) For $W > 2100$ (m/sec) and $M_\infty < 35$,

$$\frac{\rho_e}{\rho_\infty} = \frac{\rho_e}{\rho_{e_{ideal}}} \frac{\rho_{e_{ideal}}}{\rho_\infty}, \tag{16}$$

where

$$\frac{\rho_{e_{ideal}}}{\rho_\infty}$$

is given by one of the two following relations depending on $M_\infty$:

$$\text{(a)} \quad \frac{\rho_{e_{ideal}}}{\rho_\infty} = \left(\frac{6M_\infty^2}{M_\infty^2 + 5}\right)^{7/2} \left[\left(\frac{6}{7M_\infty^2 - 1}\right)\left(1 + \frac{M_\infty^2}{5}\right)\right]^{5/2} \tag{17}$$

for $M_\infty \geqq 1$.

$$\text{(b)} \quad \frac{\rho_{e_{ideal}}}{\rho_\infty} = \left(1 + \frac{M_\infty^2}{5}\right)^{5/2} \qquad \text{for } M_\infty < 1. \tag{18}$$

The relation for the ratio of real to the ideal gas value is given by

$$\frac{\rho_e}{\rho_{e_{ideal}}} = \bar{a}_o + \bar{a}_1 M_\infty + \bar{a}_2 M_\infty^2 + \bar{a}_3 M_\infty^3 + \bar{a}_4 M_\infty^4 + \bar{a}_5 M_\infty^5, \tag{19}$$

where

$$\bar{a}_o = .269623 - 2.47746 \times 10^{-2}\ S$$

$$\bar{a}_1 = .1975914 + 7.249941 \times 10^{-3}\ S$$

$$\bar{a}_2 = -1.334415 \times 10^{-2} - 6.96885 \times 10^{-4}\ S$$

$$\bar{a}_3 = 5.06022 \times 10^{-4} + 3.2325 \times 10^{-5}\ S$$

$$\bar{a}_4 = -.832101 \times 10^{-5} - 6.34127 \times 10^{-7}\ S$$

$$\bar{a}_5 = 3.569058 \times 10^{-8} + 3.77445 \times 10^{-9}\ S$$

and $S = H/1000$.

(2) For $W \leq 2100$ (m/sec) and $M_\infty < 35$,

$$\frac{\rho_e}{\rho_{e_{ideal}}} = 1. \tag{20}$$

(3) For $M_\infty \geqq 35$,

$$\frac{\rho_e}{\rho_\infty} = 20. \tag{21}$$

C. Pressure

(1) For $W > 2100$ (m/sec),

$$\frac{P_e}{P_\infty} = \frac{P_e}{P_{e_{ideal}}} \frac{P_{e_{ideal}}}{P_\infty}, \tag{22}$$

where

$$\frac{P_{e_{ideal}}}{P_\infty}$$

is given by one of the two following relations depending on $M_\infty$:

$$\text{(a)} \quad \frac{P_{e_{ideal}}}{P_\infty} = \left(\frac{6M_\infty^2}{5}\right)^{7/2} \left(\frac{6}{7M_\infty^2 - 1}\right)^{5/2} \quad \text{for } M_\infty \geqq 1. \tag{23}$$

$$\text{(b)} \quad \frac{P_{e_{ideal}}}{P_\infty} = \left(1 + \frac{M_\infty^2}{5}\right)^{7/2} \quad \text{for } M_\infty < 1. \tag{24}$$

The relation for the ratio of real to the ideal gas value is given by

$$\frac{P_e}{P_{e_{ideal}}} = C_o + C_1 M_\infty + C_2 M_\infty^2 + C_3 M_\infty^3 + C_4 M_\infty^4 + C_5 M_\infty^5, \tag{25}$$

where

$$C_o = 1.0099865 - 2.722132 \times 10^{-6} H$$

$$C_1 = -4.9934 \times 10^{-3} + .83986 \times 10^{-6} H$$

$$C_2 = 19.39623 \times 10^{-4} - .860322 \times 10^{-7} H$$

$$C_3 = -13.93188 \times 10^{-5} + .419534 \times 10^{-8}\ H$$

$$C_4 = 4.116558 \times 10^{-6} - .97149 \times 10^{-10}\ H$$

$$C_5 = -4.41709 \times 10^{-8} + .856262 \times 10^{-12}\ H.$$

(2) For $W \leqq 2100$ (m/sec),

$$\frac{P_e}{P_{e_{ideal}}} = 1. \tag{26}$$

D. The Nondimensional Velocity Gradient

(1) For $W > 2100$ (m/sec) and $M_\infty < 35$,

$$K_m = h_o + h_1 b_u + h_2 b_u^2 + h_3 b_u^3, \tag{27}$$

where

$$b_u = \frac{W}{1000} - 5 \tag{28}$$

and

$$h_o = -.1325 \times 10^{-2}\ S + .868$$

$$h_1 = -3.98 \times 10^{-4}\ S - .09081$$

$$h_2 = 1.2245 \times 10^{-4}\ S + .022684$$

$$h_3 = -1.0185 \times 10^{-5}\ S - 1.637 \times 10^{-3}.$$

(2) For $W \leqq 2100$ (m/sec) and $M_\infty < 35$,

$$K_m = 1.05. \tag{29}$$

(3) For $M_\infty \geqq 35$,

$$K_m = .63. \tag{30}$$

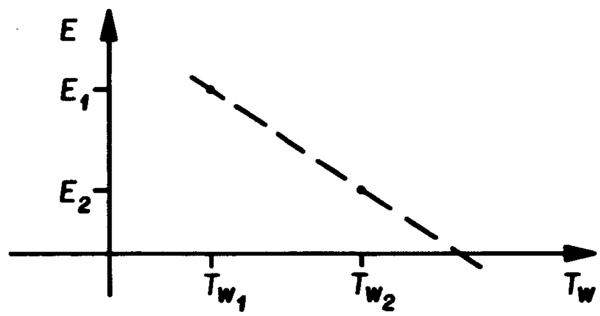
## VI. THE SOLUTION OF THE ABLATION PROBLEM

### A. The Iteration Procedure to Determine the Wall Temperature

The surface temperature, $T_w$, is determined by an iteration procedure which is satisfied when the heat balance equation at the surface-air interface is satisfied. The first guess at time t for $T_w$ is

$$T_w^{(1)}(t) = T_w(t - \Delta t) + \left(\frac{\partial T(t - \Delta t)}{\partial t}\right)_w \cdot \Delta t. \qquad (31)$$

It is desired to find a value of $T_w$ which will satisfy the heat balance equation (67). After at least two values of $T_w$ are tried and do not satisfy the relationship, one can get a good approximation of the value of $T_w$ by plotting $T_w$ vs E, where E is the error in equation (67), and finding the point where the error would be zero.




It is not necessary that the error be of opposite sign, not is it necessary to use a higher order interpolation, since the first guess will be very good and the second guess will yield an E of opposite sign or closer to zero. This can be done because the left-hand side of (67) monotonically increases and the right-hand side of (67) monotonically decreases with an increase in the size of $T_w$. By putting a straight line $T_w = a + bE$ which goes through the points $E_1$ and $E_2$, we can get the next guess for $T_w$ by evaluating the equation $T_w = a + bE$ for $E = 0$. This reduces the problem to finding "a" since $T_w = a$ for $E = 0$. Using Kramer's rule,

$$T_{w_1} = a + bE_1 \qquad a = \frac{T_{w_1} E_2 - T_{w_2} E_1}{E_2 - E_1}.$$
$$T_{w_2} = a + bE_2$$

For subsequent iterations, the last two points will be used. Care must be taken to avoid letting $E_2 - E_1$ become too small, causing overflow. The program includes a test such that a guessed value of $T_W$ would never yield a vapor pressure $P_{vap}(T_W,t)$ larger than the pressure in the boundary layer $P_e(t)$.

## B. Calculation of the Aerodynamic Heat Flux

For given values of the surface temperature, flight altitude, and flight velocity, relations are given for the aerodynamic heat flux to a nonvaporizing wall $\bar{q}_{aero}$ for the different flight regimes. The flight regimes, in the order in which an object entering the earth's atmosphere encounter, are the free molecule, transition, slip, and continuum. It was found that formula (32), given below for the aerodynamic heat flux employed by Reference 3 was a good approximation from the initial time when the object is in the free molecular regime through the transitional regime. The altitude, $H_T$, at which the object enters the slip flow regime, i.e., passes from the transition regime to the slip flow regime, is assumed to occur when the Knudsen Number (Kn) reaches a value of 0.1. Figure 5 presents the altitude $H_T$ as a function of the body radius for the earth's atmosphere.

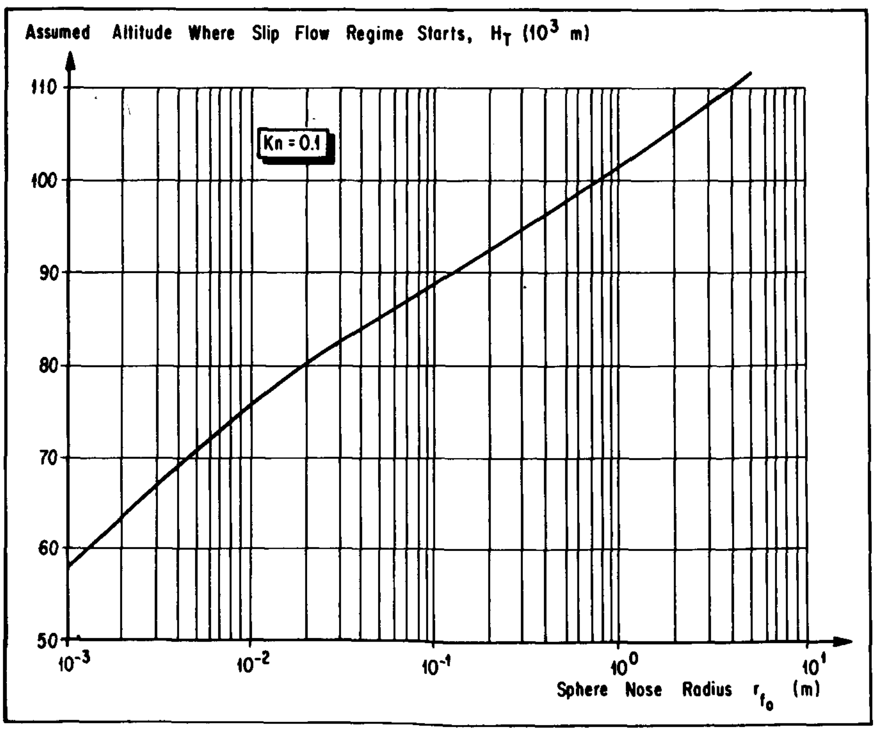



Figure 5. Altitude Where Slip Flow Regime Starts As A Function of Nose Radius

The aerodynamic heat flux $\bar{q}_{aero}$ for the slip flow regime is approximated by a smooth transition from the transitional regime, equation (32), to the continuum flow regime, equation (34). This transition takes place over a prescribed number of time steps beginning at the time when the altitude $H = H_T$.

Relations for the aerodynamic heat flux taken from various sources for the different flight regimes are as follows:

(1) For $150{,}000 \leq H(m) \leq H_T$ - Reference 3 found the following equation to be a good analytical representation of the experimental data of Reference 6 in the molecular and transitional flow regimes:

$$\bar{q}_{aero} = \frac{\bar{q}_{\substack{aero\\mol}} \; \bar{q}_{\substack{aero\\cont}}}{\sqrt{\bar{q}^2_{\substack{aero\\mol}} + \bar{q}^2_{\substack{aero\\cont}}}} = \bar{q}_{\substack{aero\\Tran}}, \tag{32}$$

where $\bar{q}_{\substack{aero\\mol}}$ is the aerodynamic heat flux for the free molecule regime given in Reference (11) for a sphere as

$$\bar{q}_{\substack{aero\\mol}} = .53733 \times 10^{-4} \rho_\infty W \left[ W^2 - 1.9262 \times 10^{+3} (T_w - T_\infty) \right] \quad (\mathrm{kcal/m^2\ sec}), \tag{33}$$

where $\rho_\infty = \rho_\infty$ (kg/m$^3$), $W = W$ (m/sec), and $T = T$ (°K). $\bar{q}_{\substack{aero\\cont}}$ is the aerodynamic heat flux in the continuum flow regime given by Reference 5 as

$$\bar{q}_{\substack{aero\\cont}} = 23{,}812.9 \sqrt{\frac{\rho_\infty}{r_f(t)}} \; (W/W_c)^{3.15} \left( \frac{h_e - h_w}{h_e - 73} \right) \quad (\mathrm{kcal/m^2\ sec}), \tag{34}$$

where

$$h_e = \frac{W^2}{8374.88} + .24\ T_\infty \qquad (\text{kcal/kg}) \tag{35}$$

$$W_c = \sqrt{\frac{397.92346 \times 10^{12}}{6.37 \times 10^6 + H}} \qquad (\text{m/sec}), \tag{36}$$

and the enthalpy at the wall $h_w$ for air is approximated by

$$h_w = -5.3944 + .24019\ T_W + 2.03371 \times 10^{-5}\ T_W^2 - 2.30515 \times 10^{-9}\ T_W^3$$

$$+ .96954 \times 10^{-13}\ T_W^4 \qquad (\text{kcal/kg}). \tag{37}$$

(2) Transition Regime from Slip Flow to Continuum Flow - At the first time t, when $H \leqq H_T$ which is designated $t_1^*$, the following transition equation for $\bar{q}_{aero}$ is introduced. It is completed in ten time steps; thus, ten time steps after $H \leqq H_T$, it is assumed that the re-entering object is in continuum flow. Let $t_1^* + 10\ \Delta t = t_2^*$.

$$\bar{q}_{aero} = \bar{q}_{\substack{aero\\tran}} + \left[\bar{q}_{\substack{aero\\cont}} - \bar{q}_{\substack{aero\\tran}}\right]\left[\frac{t - t_1^*}{t_2^* - t_1^*}\right]. \tag{38}$$

(3) From the time $t_2^*$ until time $t_3^*$ ($t_3^*$ is defined as the first time that the relation $W \leqq 2100$ (m/sec) is violated), the continuum flow relation for $\bar{q}_{aero}$ is used; i.e.,

$$\bar{q}_{aero} = \bar{q}_{\substack{aero\\cont}}.$$

(4) From the time $t_3^*$ until impact time, the following relation for the aerodynamic heat flux which was taken from Reference 13 is used:

$$\bar{q}_{aero} = \frac{k_{w,air}}{\sqrt{\mu_{w,air}}} (T_e - T_w) \sqrt{\frac{T_e \ \rho_e}{T_w}} \ (Nu/\sqrt{Re})_{Pr=1} \ (.715) \cdot \sqrt[4]{\frac{1.05W}{2r_f(t)}} \quad (39)$$

$$(kcal/m^2 \ sec).$$

The thermal conductivity of air at the wall $k_{w,air}$ is approximated by

$$k_{w,air} = .6716646 \times 10^{-8} + .2429834 \times 10^{-7} \ T_w - 1.811997 \times 10^{-11} \ T_w^2$$
$$+ 1.3873689 \times 10^{-14} \ T_w^3 - .5989437 \times 10^{-17} \ T_w^4 + 1.0229805 \times 10^{-21} \ T_w^5$$
$$[kcal/(m°K \ sec)], \quad (40)$$

and the viscosity of air at the wall $\mu_{w,air}$ is given by the Sutherland law:

$$\mu_{w,air} = \sqrt{T_w} \ \frac{14.76303}{1 + \frac{110}{T_w}} \times 10^{-7} \quad (kg/m \ sec). \quad (41)$$

The relation for $(Nu/\sqrt{Re})_{Pr=1}$ is approximated by the following curve fits of results in Reference 13:

(a) For $\frac{T_w}{T_e} < 1.6$,

$$(Nu/\sqrt{Re})_{Pr=1} = .705 + .055 \ \frac{T_w}{T_e}. \quad (42)$$

(b) For $\frac{T_w}{T_e} \geqq 1.6$,

$$(Nu/\sqrt{Re})_{Pr=1} = .755 + .025 \frac{T_w}{T_e} . \tag{43}$$

C. Heat Blockage Factor

Equations (44) - (48) in this and the next section are applicable to the continuum gas dynamic regime. In the free molecule flow regime, there is no heat blocking effect by the vaporizing species; i.e., $\psi = 1$. These equations were employed at all times in the computer program; however, no significant vaporization will generally result before the continuum flow regime is entered. The heat blockage factor, according to Reference 2, due to vaporization is

$$\psi = \frac{1 - C_{w,eq.}}{1 - C_{w,eq.} \left[1 - .68 \ (M/M_{vap})^{.26}\right]}, \tag{44}$$

where $C_{w,eq.}$, the equilibrium mass fraction of the injected vapor at the wall, is given by

$$C_{w,eq.} = \frac{1}{1 + \frac{M}{M_{vap}} \left(\frac{P_e}{P_{vap}} - 1\right)}. \tag{45}$$

M is the molecular weight of air, a function of altitude, and $M_{vap}$ is the molecular weight of the vaporizing gas. The equation for the vapor pressure $P_{vap}$ is given in the section on material properties.

D. Ablation Rate at the Wall

The ablation rate at the wall, $v_w$, is due to the vaporization process only because the melting component of ablation is zero at the wall. By combining the boundary layer solution for $C_w$ given in Reference 2 for a Lewis number of one with Scala's [16] kinetic theory expression for $C_w$, the following equation for $v_w$ results:

$$v_w = \frac{1}{\rho}\,\frac{1 - C_{w,eq} + a_{vm}\left(\frac{\psi\bar{q}_{aero}}{h_e - h_w}\right) - \sqrt{\left[1 - C_{w,eq} + a_{vm}\left(\frac{\psi\bar{q}_{aero}}{h_e - h_w}\right)\right]^2 + 4a_{vm}C_{w,eq}\left(\frac{\psi\bar{q}_{aero}}{h_e - h_w}\right)}}{2a_{vm}}, \tag{46}$$

where $a_{vm}$ is the resistance of the material to the vaporization process. It is given by Scala [16] as

$$a_{vm} = \frac{\sqrt{2\pi \mathcal{R} M_{vap}\, T_w}}{\alpha_v \left[(P_e - P_{vap})\, M + P_{vap}\, M_{vap}\right]}, \tag{47}$$

where $\mathcal{R}$ (universal gas constant) = $8.314 \times 10^3$ kg $m^2$/(kg mole $sec^2$ °K) and $\alpha_v$ is the vaporization coefficient. When $a_{vm} = 0$, the following equation for $v_w$ results:

$$v_w = -\frac{1}{\rho}\left(\frac{\psi\bar{q}_{aero}}{h_e - h_w}\right)\left(\frac{C_{w,eq}}{1 - C_{w,eq}}\right). \tag{48}$$

The computer program interprets a zero input value for $\alpha_v$ to mean $\alpha_v = \infty$ which due to (47) makes $a_{vm} = 0$; thus, when $\alpha_v = 0$ is an input, $v_w$ is computed by equation (48).

## E. The Temperature Profile

(a) The surface temperature, $T_w$, is given by an iteration process, Section VI-A.

(b) The forward difference procedures gives the temperature profile for $2\Delta Y \leqq Y \leqq (L - 1)\,\Delta Y$:

$$T(Y,t) = T(Y,t - \Delta t) + \frac{\partial T(Y,t - \Delta t)}{\partial t} \cdot \Delta t, \tag{49}$$

where $L\Delta Y$ is the thickness of the body along the axis of the symmetric body.

The temperature at the last station $L\Delta Y$ is given by

$$T(L\Delta Y, t) = T[(L - 1) \cdot \Delta Y, t]. \tag{50}$$

(c) The temperature at $Y = \Delta Y$ is given by the following curve fit:

$$T(\Delta Y, t) = \frac{1}{3} T(o, t) + T(2\Delta Y, t) - \frac{1}{3} T(3\Delta Y, t). \tag{51}$$

## F. The Ablation Rate Profile

After having a complete temperature profile, the ablation rate profile can be calculated by the following equation which results from the continuity, momentum, and the wall ablation rate equations (see Appendix A):

$$v(Y,t) = v_w(t) - \int_o^Y \int_Y^{Y_o} \frac{\left\{\frac{4}{r_f^2(t)} [P_e(t) - P_\infty(t)] + \frac{2\rho}{r_f(t)} \frac{dW}{dt}\right\} Y + 2 \frac{\tau_w}{X}}{\mu(Y,t)} \, dYdY$$

$$(\mathrm{m/sec}), \tag{52}$$

where $Y_o$ is a point in the body where the integrand is approximately zero; i.e., the material is in a solid state at $Y_o$. The viscosity of the material $\mu(Y,t)$ is given in the section on material properties as a function of the temperature $T(Y,t)$.

The shearing stress relation is given by

$$\frac{\tau_w}{X} = (\tau_w/X)_o \; [\psi(1 + \beta\psi)], \tag{53}$$

where

$$\beta = \frac{\beta_1}{(P_e/P_{vap}) - 1} \tag{54}$$

and $\beta_1$ = constant. The shearing stress at the wall for the nonvaporizing case, $(\tau_w/X)_o$, is given for a sphere by Reference 12 as

$$(\tau_w/X)_o = .727 \left[\frac{W}{r_f(t)}\left(1 - \frac{\rho_\infty}{\rho_e}\right)\right]^{1.5} \left[\frac{8}{3}\frac{\rho_\infty}{\rho_e}\right]^{.75} \left[\frac{\mu_e}{\mu_w}\frac{T_w}{T_e}\right]^{.447} \left[2\rho_e\,\mu_w\,\frac{T_e}{T_w}\right]^{.5}$$

$$(\mathrm{kg/m^2\ sec^2}), \qquad (55)$$

where $\mu_i (i = e,w)$ is given by the Sutherland law

$$\mu_i = \sqrt{T_i}\,\frac{14.76303 \times 10^{-7}}{1 + \dfrac{110}{T_i}} \qquad (\mathrm{kg/m\ sec}). \qquad (56)$$

## G. The Energy Equation

The derivative of the temperature with respect to time can now be calculated.

(1) For Y = 0 and Y = $1\Delta Y$, $\partial T/\partial t$ is approximated by a backwards difference quotient.

$$\frac{\partial T(Y,t)}{\partial t} = \frac{T(Y,t) - T(Y,t - \Delta t)}{\Delta t}. \qquad (57)$$

(2) For $2\Delta Y \leqq Y \leqq L\Delta Y$,

$$\frac{\partial T(Y,t)}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T(Y,t)}{\partial Y^2} - v(Y,t)\frac{\partial T(Y,t)}{\partial Y} - \frac{1}{\rho c_p}\frac{\partial F(Y,t)}{\partial Y}, \qquad (58)$$

where the flux of radiative energy, F, is defined by equation (A-11). For the case without internal radiation, $\partial F/\partial Y = 0$. The second derivatives of T with respect to Y in equation (58) are given by the following relations:

(a) For $Y = 0$ and $Y = 1\Delta Y$,

$$\frac{\partial^2 T(Y,t)}{\partial Y^2} = \frac{1}{k}\left[\rho c_p \left(\frac{T(Y,t) - T(Y,t-\Delta t)}{\Delta t} + v(Y,t)\,\frac{\partial T(Y,t)}{\partial Y}\right) + \frac{\partial F(Y,t)}{\partial Y}\right]. \tag{59}$$

(b) For $2\Delta Y \leqq Y \leqq (L - 1)\,\Delta Y$,

$$\frac{\partial^2 T(Y,t)}{\partial Y^2} = \frac{T(Y + \Delta Y,t) - 2T(Y,t) + T(Y - \Delta Y,t)}{(\Delta Y)^2}, \tag{60}$$

and the first derivatives of T with respect to Y are given by the following:

(a) For $Y = 0$, given in Section VI-H of this paper.

(b) For $1\Delta Y \leqq Y \leqq (L - 1)\,\Delta Y$,

$$\frac{\partial T(Y,t)}{\partial Y} = \frac{T(Y + \Delta Y,t) - T(Y - \Delta Y,t)}{2\Delta Y}. \tag{61}$$

At the last grid point $L\Delta Y$, both the first and second derivatives of T with respect to Y are zero.

## H. The Heat Balance Equation

A heat balance equation at the gas-liquid interface which must be satisfied at each time t is now derived, and is based on the fact that no heat can be stored at the gas-liquid interface. The net flux of heat on the gas side of the interface must equal the net flux of heat on the liquid side. Hypersonic entry into the earth's atmosphere of a blunt-nosed object generates a curved detached shock wave which results in very large (gas-cap) temperatures. The radiation, termed here gas-cap radiation, from the high temperature air is approximately proportional to the nose radius of the object. This heat addition is neglected in the method presented herein since we were concerned only with objects of very small radius such as tektite and australite

bodies. However, gas-cap radiation should be accounted for when larger bodies are being considered. On the gas side, there are the following amounts of heat either arriving at or leaving the interface:

(1) the aerodynamic heat flux ($\bar{q}_{aero}\ \psi$) arrives at the interface,

(2) the heat flux taken up by the vaporization process, ($q_{vap} = -\rho v_w\ h_v$) leaving the interface, and

(3) the heat flux radiated ($q_{rad}$) which leaves the interface.

The heat fluxes on the liquid side of the interface are

(1) the heat flux conducted at the interface, $q_c = -k(\partial T/\partial Y)_w$, and

(2) the heat flux being radiated up to the interface from the interior of the body, ($q_{rad}$); this term is zero for the case without internal radiation.

Equating the net heat flux on the gas side of the interface with the net heat flux on the liquid side yields

$$\bar{q}_{aero}\ \psi - q_{vap} - q_{rad} = q_c - q_{rad}, \tag{62}$$

which can be written as

$$\bar{q}_{aero}\ \psi + \rho v_w\ h_v = -k(\partial T/\partial Y)_w. \tag{63}$$

For the case without internal radiation, this equation is

$$\bar{q}_{aero}\ \psi + \rho v_w\ h_v - q_{rad} = -k(\partial T/\partial Y)_w, \tag{64}$$

where $q_{rad}$ is given by Stefan's law for emission of energy from the surface of a body:

$$q_{rad} = \epsilon\,\sigma\,T_w^4, \tag{65}$$

where $\sigma = 1.378 \times 10^{-11}$ [kcal/m$^2$ sec(°K)$^4$].

The heat conduction term $-k(\partial T/\partial Y)_w$ in equation (63) is determined by integrating the energy equation (A-3) over Y from $Y = 0$ to $Y = Y_B$ where $Y_B$ is any point within the body. It is best to make $Y_B$ at least equal to $6\Delta Y$ to perform the numerical integration involved to an agreeable degree of accuracy. Integrating the energy equation (A-3) yields

$$-k\left(\frac{\partial T}{\partial Y}\right)_w = -k\left(\frac{\partial T}{\partial Y}\right)_{Y_B} + \rho c_p \int_o^{Y_B} \left(\frac{\partial T}{\partial t} + v\,\frac{\partial T}{\partial Y}\right) dY + F_{Y_B} - F_w. \qquad (66)$$

Combining equations (63) and (66) yields the following equation, which is subsequently called the heat balance equation:

$$\bar{q}_{aero}\,\psi + \rho v_w\, h_v = -k\left(\frac{\partial T}{\partial Y}\right)_{Y_B} + \rho c_p \int_o^{Y_B} \left(\frac{\partial T}{\partial t} + v\,\frac{\partial T}{\partial Y}\right) dY + F_{Y_B} - F_w. \qquad (67)$$

For the case without internal radiation the heat balance equation is

$$\bar{q}_{aero}\psi + \rho v_w\, h_v - q_{rad} = -k\left(\frac{\partial T}{\partial Y}\right)_{Y_B} + \rho c_p \int_o^{Y_B} \left(\frac{\partial T}{\partial t} + v\,\frac{\partial T}{\partial Y}\right) dY. \qquad (68)$$

The iteration for $T_W$ is ended when the heat balance equation is satisfied within a prescribed tolerance.

## I. Calculation of the Mass m(t)

The equations presented in this section are based on the body shapes given in Figures 2 and 3.

### (1) Mass for Method 1 Geometry (see Figure 2)

For this method, it is assumed that all of the material that melts and vaporizes is removed from the body. The equation for the mass of the body which varies with time due to the ablation processes is

$$m(t) = m_o - \rho\pi \left[ r_{f_o}^2 Y_s - \frac{r_{f_o} Y_s^2}{2} + \frac{Y_s^3}{6} \right] \tag{69}$$

where $m_o$ is the initial mass of the body.

(2) Mass for Method 2 Geometry (see Figure 3)

For this method it is assumed that

(a) mass lost = mass lost due to vaporization and

(b) mass melted = mass of the two flanges.

The points ($\xi_1$ $\eta_1$) shown in Figure 3 are calculated by

$$\xi_1 = \frac{r_{f_o}^2 - r_f^2(t) + h^2}{2h} \tag{70}$$

and

$$\eta_1 = \pm \sqrt{r_{f_o}^2 - \frac{[r_{f_o}^2 - r_f^2(t) + h^2]^2}{4h^2}}, \tag{71}$$

where

$$h = r_f(t) - r_{f_o} + Y_s \tag{72}$$

and $Y_s$ is given by equation (2). The radius $r_{f_v}$ is given by

$$r_{f_v} = \frac{1}{2} (\xi_1 + \delta_1) + \frac{\eta_1^2}{2(\xi_1 + \delta_1)} \tag{73}$$

$$\delta_1 = r_{f_o} - Y_{s_w}, \tag{74}$$

where $Y_{s_w}$ is given by equation (4).

$$j = r_{f_v} - \delta_1. \tag{75}$$

The radius $r_f(t)$ is given by equation (3). The volume lost due to vaporization is given by

$$\Delta V_{vap} = \pi \left\{ r_{f_o}^2 (\xi_1 + r_{f_o}) - r_{f_v}^2 (\xi_1 + \delta_1) - \frac{1}{3} \left[ \xi_1^3 + r_{f_o}^3 - (\xi_1 - j)^3 - (\delta_1 + j)^3 \right] \right\}. \tag{76}$$

The mass of the body at time t is then given by

$$m(t) = m_o - \rho \Delta V_{vap}, \tag{77}$$

where $m_o$ is the initial mass of the body.

## VII. PHYSICAL PROPERTIES OF THE BODY MATERIAL

Physical properties of the glassy material pertinent to the ablation problem are the following:

(a) The thermal conductivity, k. (kcal/m°K sec).

(b) The density, $\rho$. (kg/m$^3$).

(c) The specific heat at constant pressure, $c_p$. (kcal/kg°K).

(d) The emissivity constant at the surface, $\epsilon$. (-)

(e) The molecular weight of the vaporized gas, $M_{vap}$. (kg/kg mole).

(f) The heat of vaporization, $h_v$. (kcal/kg).

(g) The viscosity, $\mu$ (kg/m sec) represented by the function

$$\mu = B_1 \exp \left[ \frac{B_2}{T - B_3} + B_4 \right], \quad \text{(kg/m sec)} \tag{78}$$

where $B_1$, $B_2$, $B_3$, and $B_4$ are constants.

(h) The vapor pressure $P_{vap}$ (kg/m sec$^2$) represented by the following function recommended by Chapman [3] which accounts for vaporization suppression by oxygen:

$$P_{vap} = \frac{P_e}{(P_e/P^*_{vap})^{m_1}}, \quad (\text{kg/m sec}^2) \tag{79}$$

where $m_1$ = constant. The equilibrium vapor pressure of the vaporized gas $P^*_{vap}$ is given by

$$P^*_{vap} = A_1 \exp\left[\frac{A_2}{T_w^2} + \frac{A_3}{T_w} + A_4\right], \quad (\text{kg/m sec}^2) \tag{80}$$

where $A_1$, $A_2$, $A_3$, and $A_4$ are constants.

(i) The refractive index, n. (-)

(j) The absorption coefficient, $\alpha_A$. (1/m).

(k) The reciprocal radiation mean free path, $\alpha$. (1/m).

(l) The effective reflectivity of the surface, $R_{eff}$. (-).

## APPENDIX A

### The Differential Equations of the Glass Layer

The three differential equations describing the viscous glass layer in the vicinity of the stagnation point are well known. The equations of continuity, momentum, and energy, where the reference system is fixed at the stagnation point, with independent variables X, Y measured in the direction shown in Figure 6 with the corresponding velocity components are as follows:

Continuity

$$\frac{u}{X} + \frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} = 0. \tag{A-1}$$

Momentum

$$\frac{\partial}{\partial Y}\left(\mu \frac{\partial u}{\partial Y}\right) = \frac{dP}{dX} - \frac{\rho X}{r_f}\frac{dW}{dt} . \tag{A-2}$$

Energy

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial Y^2} - \rho c_p \, v \frac{\partial T}{\partial Y} - \frac{\partial F}{\partial Y}. \tag{A-3}$$

Because of the large Reynolds number of the glass-liquid layer, the inertia terms have been omitted, since they are negligible in comparison to the shear and pressure gradient terms. The variation of the pressure in the Y direction is assumed to be zero which is consistent with usual boundary layer equations.

In the vicinity of the stagnation point the velocity component u varies linearly with X; i.e., u = cX. The continuity equation (A-1) can therefore be written as

$$u = - \frac{X}{2}\frac{\partial v}{\partial Y} , \tag{A-4}$$

which when differentiated with respect to Y yields

$$\frac{\partial u}{\partial Y} = - \frac{X}{2} \frac{\partial^2 v}{\partial Y^2} . \tag{A-5}$$

Integrating the momentum equation from the surface Y = 0 to an arbitrary point in the glass-liquid layer yields:

$$\mu \frac{\partial u}{\partial Y} = \left[\frac{dP}{dX} - \frac{\rho X}{r_f} \frac{dW}{dt}\right] Y - \tau_w \tag{A-6}$$

where

$$\tau_w = - \left(\mu \frac{\partial u}{\partial Y}\right)_w . \tag{A-7}$$

Substituting equation (A-5) into (A-6) yields

$$\frac{\partial^2 v}{\partial Y^2} = \frac{\left[- \frac{2}{X} \frac{dP}{dX} + \frac{2\rho}{r_f} \frac{dW}{dt}\right] Y + 2 \frac{\tau_w}{X}}{\mu} . \tag{A-8}$$

The Newtonian pressure distribution yields the pressure term in the vicinity of the stagnation point as

$$\frac{1}{X} \frac{dP}{dX} = - \frac{2}{r_f^2} (P_e - P_\infty). \tag{A-9}$$

Integration of equation (A-8) twice, first from $Y_o$ to Y and then from zero to Y and due to equation (A-9) and the boundary condition $\partial v/\partial Y = 0$ at $Y = Y_o$ results in the following equation for the ablation velocity v

$$v(Y,t) = v_w(t) - \int_o^Y \int_Y^{Y_o} \frac{\left\{\frac{4}{r_f^2(t)}\left[P_e(t) - P_\infty(t)\right] + \frac{2\rho}{r_f(t)} \frac{dW(t)}{dt}\right\} Y + 2 \frac{\tau_w(t)}{X}}{\mu(Y,t)} dYdY. \tag{A-10}$$

The flux of radiative energy represented by the symbol F which appears in the energy equation (A-3) is given by (see Kadanoff [9])

$$\frac{F}{2n^2\alpha_A\sigma} = \int_0^Y T^4(\eta)\ \exp\left[-\alpha(Y - \eta)\right] d\eta - \int_Y^\infty T^4(\eta)\ \exp\left[-\alpha(\eta - Y)\right] d\eta$$

$$+ R_{eff} \int_0^\infty T^4(\eta)\ \exp\left[-\alpha(Y + \eta)\right] d\eta \tag{A-11}$$

where n is the refractive index, $\alpha_A$ is the absorption coefficient, $\sigma$ is the Stefan-Boltzmann constant, $\alpha$ is the reciprocal radiation mean free path, and $R_{eff}$ is the effective reflectivity of the surface.

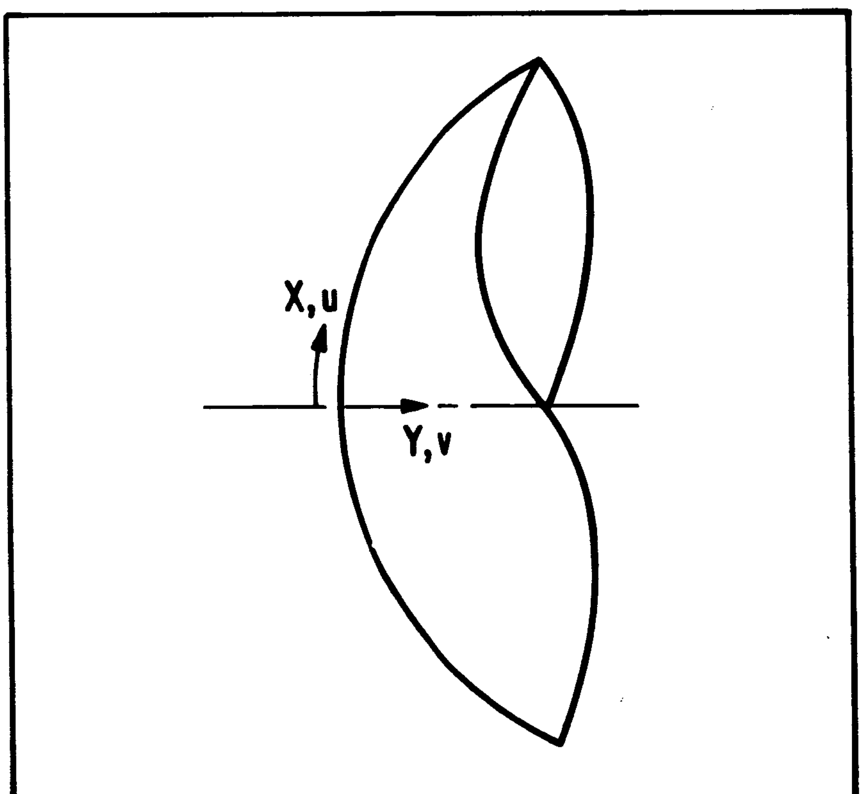



Figure 6. Space Variables (X,Y) and Corresponding Velocity Variables for Surface Fixed Reference System

APPENDIX B

The Fortran Program and Its Input Data

## A. Preparation of the Input Data for the Computer Program

The computer program and the subroutines employed by the program are presented in Fortran IV language in Part B of this Appendix. To run this program, the following input cards must be prepared:

| Card No. | Columns | |
|---|---|---|
| 1 | 1 | Integer denoting the method (Method 1 or Method 2) to be used for calculating the front face radius and mass of the body. Should be 1 for Method 1 and 2 for Method 2. I1 format. |
| | 2-3 | Not used. |
| | 4 | This column is used to give alternative of whether to include internal radiation effects (with internal radiation) or assume glass is opaque and account for only heat flux radiated away from the surface (without internal radiation). Should be 1 for case without internal radiation or 2 for case with internal radiation. I1 format. |
| | 5-16 | Not used. |
| | 17-32 | Initial body shape, sphere or hemisphere; should be 0 for hemisphere, non-zero for sphere. |
| | 33-48 | $\Delta Y$ (thickness grid along axis). |
| | 49-64 | $r_{f_o}$ (initial front face radius). |
| 2 | 1-16 | $H_o$ (initial altitude). |
| | 17-32 | $W_o$ (initial velocity). |
| | 33-48 | $\phi_o$ (initial entry angle in degrees). |
| | 49-64 | $H_T$ (altitude where slip flow regime begins, see Figure 5). |

| Card No. | Columns | |
|---|---|---|
| 3 | 1-16 | k (thermal conductivity of body material). |
| | 17-32 | $\rho$ (density of body material). |
| | 33-48 | $c_p$ (specific heat of body material). |
| | 49-64 | $M_{vap}$ (molecular weight of vapor). |
| | 65-80 | $h_v$ (heat of vaporization of body material). |
| 4 | 1-16 | $\alpha_v$ (vaporization coefficient). |
| | 17-32 | $\beta_1$ (constant in shear stress relation). |
| 5 | 1-16 | $A_1$ (constant in equilibrium vapor pressure function). |
| | 17-32 | $A_2$ (constant in equilibrium vapor pressure function). |
| | 33-48 | $A_3$ (constant in equilibrium vapor pressure function). |
| | 49-64 | $A_4$ (constant in equilibrium vapor pressure function). |
| | 65-80 | $m_1$ (constant in vapor pressure relation). |
| 6 | 1-16 | $B_1$ (constant in viscosity function). |
| | 17-32 | $B_2$ (constant in viscosity function). |
| | 33-48 | $B_3$ (constant in viscosity function). |
| | 49-64 | $B_4$ (constant in viscosity function). |

| Card No. | Columns | |
|---|---|---|
| 7 | 1-16 | $\epsilon$ (emissivity constant of opaque body material). |
| | 17-32 | $\alpha$ (reciprocal radiation mean free path). |
| | 33-48 | $\alpha_A$ (absorption coefficient). |
| | 49-64 | n (refractive index). |
| | 65-80 | $R_{eff}$ (effective reflectivity of the surface). |
| 8 | 1-16 | $\Delta t_1$ (first time step). |
| | 17-32 | $Tm_1$ (first maximum time). |
| | 33-48 | $Mp_1$ (print frequency for first interval; I4 format, right adjusted to column 36). |
| 9 | 1-16 | $\Delta t_2$ (second time step). |
| | 17-32 | $Tm_2$ (second maximum time). |
| | 33-48 | $Mp_2$ (print frequency for second interval; I4 format, right adjusted to column 36). |
| 10 | 1-16 | $\Delta t_3$ (third time step). |
| | 17-32 | $Tm_3$ (third maximum time). |
| | 33-48 | $Mp_3$ (print frequency for third interval; I4 format, right adjusted to column 36). |

Unless specifically stated otherwise, all fields are formated E16.8.

B. THE FORTRAN PROGRAM AND SUBROUTINES

```
C      THE ABLATIØN PRØGRAM WITH INTERNAL RADIATIØN ØPTIØN
C         FØR INTERNAL RADIATIØN,SET CØL. 4 ØF 1ST INPUT CARD =2
C         SET CØL. 4 ØF 1ST INPUT CARD = 1 IF NØ INTERNAL RADIATIØN
C           IF  INTERNAL  RAD ØPTIØN IS USED,AN EXTRA INPUT CARD
C             IS REQUIRED,WHICH GIVES THE RADIATIØN CØNSTANTS
C                THIS BECØME THE 9TH INPUT CARD.
      ABSF(X)=ABS(X)
      DIMENSIØN  RG(100), RG1(100),TD(100),Z(100)
      REAL MUE,KM,MSTAR,M22,M1,K,KWAIR,LH,MXTIM,MUWAIR,MU,MVAP,
     1MØØ,MASS,MØ,MUINF,M,MUW
      EQUIVALENCE (RFØ,RFO),(VØØ,VØO,VOØ,VOO),(TLIMT(2),TMLMT(1)),
     1(MØØ,MØO,MOØ,MOO),(PØØ,PØO,POØ,POO),(MØ,MO),(TØØ,TØO,TOØ,TOO,TINF)
     1,(E60SEC,E6ØSEC),
     2(PS,PE),(TS,TE),(RS,RHØE,RHOE),(RHØINF,RHOINF),(AØØ,AØO,AOØ,AOO),
     3(TØ,TO),(VI,VW)
      EQUIVALENCE (MUWAIR,MUAIR)
      DIMENSIØN IPRT(3)
      DIMENSIØN TMLMT(3)
      DIMENSIØN TLIMT(4)
      DIMENSIØN RA(100),RA1(100),DFDY(100)
      EQUIVALENCE (TLIMT(2),TMLMT(1))
      DIMENSIØN TDT(3),         CØFHW(5),CØFK(6),T(100),VV(100),
     1TLAST(100),D2Y(100),DTDY(100),Y(100),DTDT(100),DTDTI(100),
     2YB(200),TMU(200),ARG(200),ARG1(200)
      DIMENSIØN  PR(15)
      DIMENSIØN TABVI(3),TABVØ(3)
 3900 FØRMAT(49X33HMATERIAL PRØPERTIES AND CØNSTANTS///)
 3901 FØRMAT(10X25HK(THERMAL CØNDUCTIVITY) =23XE16.8,26X20H(KCAL/M/DEG K
     1EL/SEC))
 3932 FØRMAT(1H09X5HDY = E13.6,8H  METERS)
 3955 FØRMAT(/////)
 3903 FØRMAT(1H09X19HCP(SPECIFIC HEAT) =29XE16.8,26X16H(KCAL/KG/DEG K) )
 3902 FØRMAT(1H09X14HRHØ(DENSITY) =34XE16.8,26X10H(KG/M**3) )
 3925 FØRMAT(1H09X25HE (EMISSIVITY CØNSTANT) =23XE16.8)
 3906 FØRMAT(1H09X18HALFAV(VAP CØEFF) =30XE16.8,26X17HZERØ FØR INFINITY)
 3904 FØRMAT(1H09X23HMVAP(MØL WT ØF VAPØR) =25XE16.8,26X12H(KG/KG MØLE))
 3905 FØRMAT(1H09X26HHV(HEAT ØF VAPØRIZATIØN) =22XE16.8,26X9H(KCAL/KG))
 3907 FØRMAT(1H09X5HYB = 43XI4,3H DY35X10H(METERS)  )
 3908 FØRMAT(1H09X19HVISCØSITY FUNCTIØN /1H018X27HMU = B1*EXP(B2/(T-B3)
     1+ B4)54X10H(KG/M/SEC))
 3909 FØRMAT(1H018X5HB1 = E16.8,/1H018X5HB2 = E16.8,/1H018X5HB3 = E16.8,
     1/1H018X5HB4 = E16.8)
 3910 FØRMAT(1H09X23HVAPØR PRESSURE FUNCTIØN)
 3911 FØRMAT(1H018X25HPVAP= PE/((PE/PVAPS)**M1))
 3912 FØRMAT(1H018X34HPVAPS =A1*EXP(A2/TW**2 +A3/TW +A4)47X13H(KG/M/SEC*
     1*2))
 3913 FØRMAT(1H018X5HM1 = E16.8,/1H018X5HA1 = E16.8,/1H018X5HA2 = E16.8,
     1/1H018X5HA3 = E16.8,/1H018X5HA4 =  E16.8)
 3914 FØRMAT(1H050X29HINITIAL BØDY GEØMETRY METHØD I1,///)
 3915 FØRMAT(1H09X16HBØDY IS A SPHERE)
 3916 FØRMAT(1H09X20HBØDY IS A HEMISPHERE)
 3917 FØRMAT(1H09X22HRFØ (INITIAL RADIUS) =26XE16.8,26X8H(METERS))
 3918 FØRMAT(1H09X22HMØ  (INITIAL MASS) =  26XE16.8,26X4H(KG))
 3919 FØRMAT(1H052X29HINITIAL TRAJECTØRY CØNDITIØNS)
 3920 FØRMAT(1H018X21HH (FLIGHT ALTITUDE) =18XE16.8,26X8H(METERS))
 3921 FØRMAT(1H018X21HW (FLIGHT VELØCITY) =18XE16.8,26X7H(M/SEC))
```

```
 3922 FØRMAT(1H018X23HPHI (ANGLE ØF ATTACK) =16XE16.8,26X9H(DEGREES))
 3923 FØRMAT(1H062X9HGRID SIZE///)
 3924 FØRMAT(1H09X5HDT = E13.6,16H BETWEEN TIME = E14.6,12H AND TIME = E
     114.6,33H WITH A PRINT FREQUENCY ØF EVERY I4,11H TIME STEPS)
4941  FØRMAT(///10X35H THIS RUN WILL TERMINATE AT TIME = E16.8,4H SEC)
 6931 FØRMAT(1H09X17HALFV (VAP CØEF) =33X8HINFINITY)
C     BEGIN CALCULATIØNS
3969  FØRMAT(E16.8,E16.8,I4)
 3960 FØRMAT(5E16.8)
1     CØNTINUE
      CALL SCLØCK (DATE,CTIME,ESEC,E60SEC)
      MM=10
      TLIMT(1)=0.
      PI= 3.1415927
      G=1.4
      IYØ=6
      TØ=300.
 4444 FØRMAT(I1,2XI1,12X,3E16.8)
      READ (5,4444) METHØD,IRAD,SPHERE,DY,RFØ
C     IRAD=2 FØR INTERNAL  RADIATIØN ØPTIØN
C     IRAD =1 FØR CASE WITHØUT  RADIATIØN
      IF (IRAD) 8010,8011,8010
 8010 IF  (IRAD-2) 8013,8013,8011
C     IF IRAD NØT =1 ØR 2  SET =1
 8011 IRAD=1
 8013 CØNTINUE
      IF (METHØD-1) 2344,2345,2344
C     IF METHØD NØT EQUAL 1, SET IT EQUAL 2 AND CØNTINUE
C     METHØD DENØTES METHØD TØ CALCULATE BØDY GEØMETRY
 2344 METHØD=2
2345  CØNTINUE
      READ (5,3960) HØ,WØ,PHI,H70
      READ (5,3960) K,RHØ,CP,MVAP,HV,ALFV,SE1
      READ (5,3960) A1,A2,A3,A4,M1,B1,B2,B3,B4
      READ (5,3960) E,ALFR,ALFA,RN,REFF
      SIGMA=.1378E-10
      GØ TØ (8015,8016),IRAD
 8016 E=0.
 8015 CØNTINUE
      DØ3971 IKA=1,3
3971  READ(5,3969) TDT(IKA),TMLMT(IKA),IPRT(IKA)
      IYZ=IYØ+1
      CØNST=2.*RN**2 *ALFA*SIGMA
      KEY=1
      HMAX =HØ +50000.
      R=8314.
      AVM =0.
      MXTIM=TMLMT(1)
C     SELECT LARGEST ENTRY IN  TIME TABLE FØR END ØF JØB TIME
      DØ 3961 I=2,3
      IF (MXTIM-TMLMT(I)) 3962,3961,3961
 3962 MXTIM= TMLMT(I)
 3961 CØNTINUE
      TAWX=0.
      PHIØ=PHI*PI/180.
      MØ=4./3.*PI*RFØ**3*RHØ
      IF (SPHERE)626,625,626
625   MØ=MØ*.5
626   CØNTINUE
3944  FØRMAT(1H09X8HBETA1 = 40XE16.8)
3931  FØRMAT(1HO////)
```

```
C       READ INTERNAL CLØCK, PRINT DATE AND TIME
 3927   FØRMAT (1H1)
 3997   FØRMAT(1H1,100X6HTIME   ,A6,/101X6HDATE   ,A6,// )
        WRITE (6,3997 ) CTIME, DATE
        SEC = E60SEC
        GØ TØ (8040,8041),IRAD
 8040   CØNTINUE
 8439   FØRMAT(43X47H THE ABLATIØN PRØGRAM WITHØUT INTERNAL RADITIØN////)
        WRITE (6,8439)
        GØ TØ 8042
 8041   CØNTINUE
4939    FØRMAT(43X48H THE ABLATIØN PRØGRAM WITH INTERNAL RADIATIØN    ///)
        WRITE (6,4939)
 8042   CØNTINUE
        WRITE (6,3900)
        WRITE (6,3901) K
        WRITE (6,3902) RHØ
        WRITE (6,3903) CP
        GØ TØ (8017,8018),IRAD
 8017   CØNTINUE
        WRITE (6,3925) E
 8018   CØNTINUE
        WRITE (6,3904) MVAP
        WRITE (6,3905) HV
        IF (ALFV) 6932,6934,6932
 6934   CØNTINUE
        WRITE (6,6931)
        GØ TØ 6933
 6932   CØNTINUE
        WRITE (6,3906) ALFV
 6933   CØNTINUE
        WRITE (6,3944)SE1
        WRITE(6,3907)IYØ
        WRITE (6,3908)
        WRITE (6,3909) B1,B2,B3,B4
        WRITE (6,3910)
        WRITE (6,3911)
        WRITE (6,3912)
        WRITE (6,3913) M1,A1,A2,A3,A4
        GØ TØ (8020,8019),IRAD
C       PRINT INTERNAL RADIATIØN CØNSTANTS
 8019   CØNTINUE
 8000   FØRMAT(9X8HR(EFF) =41XE16.8)
 8001   FØRMAT(1H08K3HN =46XE16.8)
 8003   FØRMAT(1H08X8HALF   =   41XE16.8,26X10H(1/METERS))
 8002   FØRMAT(1H08X6HALFA= 43XE16.8,26X10H(1/METERS))
 8004   FØRMAT(1H08X7HSIGMA =42XE16.8,26X26H(KCAL/(M**2 SEC DEG K**4)))
 8005   FØRMAT(1H151X28HINTERNAL RADIATIØN CØNSTANTS///)
        WRITE (6,8005)
        WRITE (6,8000) REFF
        WRITE (6,8001) RN
        WRITE (6,8002) ALFA
        WRITE (6,8003) ALFR
        WRITE (6,8004) SIGMA
 8020   CØNTINUE
        WRITE (6,3927)
        WRITE (6,3914) METHØD
        IF(SPHERE)3928,3929,3928
 3929   WRITE (6,3916)
       .GØ TØ 3930
 3928   WRITE (6,3915)
```

```
 3930 CØNTINUE
      WRITE (6,3917) RFØ
      WRITE (6,3918) MØ
      WRITE (6,3955)
      WRITE (6,3919)
      WRITE (6,3920) HØ
      WRITE (6,3921) WØ
      WRITE (6,3922) PHI
      WRITE (6,3955)
      WRITE (6,3923)
C     SET UP PRINT ØF TIME STEPS AND PRINT INTERVAL.
      DØ 3111 MMM=1,2
      IF (TMLMT(MMM)-TMLMT(MMM+1)) 3111,2222,2222
 3111 CØNTINUE
      MMM=MMM+1
 2222 CØNTINUE
      DØ  3933 IKJ=1,MMM
 3933 WRITE (6,3924) TDT(IKJ), TMLMT(IKJ-1),TMLMT(IKJ),IPRT(IKJ)
      WRITE(6,3932) DY
      WRITE (6,4941) MXTIM
      HKM = H70
      H70KM =H70
      KTR=1
      IPRINT=IPRT(1)
      Q2TØL=.01
      QTØL=.001
      DY2 = DY*DY
      RKCP = K/(RHØ* CP)
      RCP=1./(RHØ*CP)
      TST= TDT (1)
      DIV=MM
      DØ 909 I=2,3
      IF (TDT(I)-TST)909,909,908
 908  TST =TDT (I)
 909  CØNTINUE
      IF (TST/DY2- 1./(RKCP*PI) )911,911,910
C     GRID RATIØ VIØLATED, DY  WILL BE CALCULATED
 910  DY2=RKCP*PI*TST
      DY=SQRT(DY2)
      WRITE(6,912)
 912  FØRMAT(43HOGRID RATIØ VIØLATED, DY WILL BE CALCULATED)
 911  CØNTINUE
      TIME=0.
      TM= 0.
      A6= 397.92346E12
      A7= 6.37E6
      A8=23812.9
      A9=3.15
      A10 = 73.
      A11 = .53733E-4
      A12 = 1926.2
      A13 = .715
      A14=.705
      A15 = .055
      A16 = .755
      A17 = .025
      A18 = 14.76303E-7
      A19 = 110.
      CØF HW(1)=.96954E-13
      CØF HW(2)=-2.30515E-9
      CØFHW(3)= 2.03371E-5
```

```
      CØFHW(4) =.24019
      CØFHW(5)=-5.3944
      CØ1=1.0099865
      CØ2=-2.722132E-6
      C11=-4.9934E-3
      C12=.83986E-6
      C21= 19.39623E-4
      C22=-.860322E-7
      C31 =-13.93188E-5
      C32 = .419534E-8
      C41 =4.116558E-6
      C42 =-.97149E-10
      C51 =-4.41709E-8
      C52 =.856262E-12
      C26 = .26
      C68 = .68
      CØFK(1) = 1.0229805E-21
      CØFK(2)= -.5989437E-17
      CØFK(3)=1.3873689E-14
      CØFK(4)=-1.811997E-11
      CØFK(5)= .2429734E-7
      CØFK(6)= .6716646E-8
      RFT=RFØ
      SI=1.
      PVAP=0.
      YS=0.
      YSW=0.
      IKI =1
      RFV=RFØ
      IF(SPHERE)3940,3941,3940
 3941 THICK =RFØ
      GØ TØ 3942
 3940 THICK =2.*RFØ
 3942 CØNTINUE
C     CALCULATE NUMBER ØF STEPS IN Y DIRECTIØN TØ TAKE CALCULATIØNS
      AT= THICK/DY +1.5
      N=AT
      IF (N-100)11,10,10
 10   N=100
 11   CØNTINUE
      DØ 12 I=1,N
      T(I) =TO
      TLAST(I)=TØ
      DTDT(I)=0.
      D2Y(I)=0.
      DTDY(I) =0.
      DFDY(I)=0.
      Y(I) = DY* FLØAT (I-1)
 12   CØNTINUE
      FØ=0.
      FYB=0.
      VI=0.
      KK=1
      DT =TDT(1)
      DT2=DT*.5
      KPRINT=0
      H=HØ
      HN=H
      W=WØ
      U= WØ* CØS (PHIØ)
      V= WØ *SIN (PHIØ)
```

```
      UN=U
      VN=V
      RSEA=6380000.
      GSEA =9.80665
      MASS= MØ
      KR= 1
      GØ TØ 500
C     END ØF  PRECØMPUTE  SECTIØN
1000  CØNTINUE
C     BEGAN CALCULATIØNS FØR A NEW TIME
      TMM =  TIME - TMLMT (KK)
      IF (KK-3) 5051,5051,696
 5051 CØNTINUE
      IF (TMM) 5052, 84,84
 5052 IF (ABS(TMM)-DT/4.) 84,13,13
C     CHANGE TIME STEP, AND PRINT FREQUENCY
 84   KK=KK+1
      IKI=1
      DT= TDT(KK)
      IPRINT=IPRT(KK)
      DT2 =DT*.5
 13   CØNTINUE
C     CALCULATE NUMBER ØF STEPS IN Y DIRECTIØN TØ TAKE CALCULATIØNS
      AT=(THICK -YS)/DY +1.5
      N=AT
      IF (N-100) 1111,1111,1110
 1110 N=100
 1111 CØNTINUE
      IF (N-IYZ) 1112,1113,1113
1114  FØRMAT(29H0BØDY MELTED AWAY,THICKNESS =I4,3H DY)
 1112 KEY=2
      WRITE (6,1114) N
      GØ  TØ 507
 1113 CØNTINUE
      KPRINT=KPRINT+1
      DØ 33 I=1,N
      T(I)=TLAST(I)+DTDT(I)*DT
33    CØNTINUE
      T(N)= T(N-1)
C     FØURTH ØRDER RUNGE KUTTA PRØCEEDURE, KR DESIGNATES THE PASS
C     KR=1,INITIAL PASS
C     KR=2,1ST PASS ØN ANY  TIME  STEP
C     KR=3,2ND PASS ØN ANY  TIME  STEP
C     KR=4,3RD PASS ØN ANY  TIME  STEP
C     KR=5,4TH PASS ØN ANY  TIME  STEP
      FK1U = DUDT*DT
      FK1V = DVDT *DT
      TIME= TM +DT2
      KR=2
      U= UN + FK1U*.5
      V= VN + FK1V*.5
      GØ TØ 500
 1001 FK2U= DUDT*DT
      FK2V = DVDT*DT
      U = UN + FK2U*.5
      V = VN + FK2V*.5
      KR= 3
      GØ  TØ 500
 1002 FK3U= DUDT*DT
      FK3V =DVDT*DT
      TIME= TM+DT
```

```
      U=  UN + FK3U
      V=  VN + FK3V
      KR=4
      GØ TØ 500
 1003 FK4U= DUDT*DT
      FK4V= DVDT*DT
      U= UN+1./6.*(FK1U +2.*(FK2U+FK3U)+FK4U)
      V= VN+1./6.*(FK1V+ 2.*(FK2V +FK3V)+FK4V)
      KR=5
      GØ TØ  500
 1004 CØNTINUE
C     END ØF RUNGE KUTTA
C     CØMPUTE  MELTING  VALUES
 100  CØNTINUE
      M22=MØØ**2
      TW =T(1)
      S=H/1000.
      IF (W-2100.) 11405,11409,11409
11405 IF (MØØ-1.) 11406,11409,11409
11406 PST1=1.+M22*.2
      RE1=PST1**2.5
      PSP=RE1*PST1
      GØ TØ 11408
11409 CØNTINUE
      PSP=      (6./5.* MØØ**2)**3.5 * (6./(7.*MØØ**2-1.))**2.5
      RE1 = ((6.*M22)/(M22+5.))**3.5 * ( 6./(7.*M22-1.)* (1.+M22/5.))
     1**2.5
11408 CØNTINUE
      PSID=PSP* PØØ
      TEIDAL= TØØ*(1.+M22/5.)
      DWDT=(U*DUDT+V*DVDT)/W
      CØN= .5*RHØ*DWDT
      CØ= CØ1 +CØ2*H
      C1= C11+C12*H
      C2= C21+ C22*H
      C3= C31+ C32*H
      C4= C41+ C42*H
      C5= C51 + C52*H
      PSPS=           ((((C5*MØØ+C4)*MØØ+C3)*MØØ+C2)*MØØ+C1)*MØØ+CØ
      PS= PSPS*PSID
      IF (MØØ -35.)1401,1402,1402
1402  KM=.63
      RHØE = 20.*RHØINF
      TE = 45.* TØØ
      GØ TØ 1405
1401  IF (W-2100.) 1403,1403,1404
C     CALCULATE TE, RHØE,KM AND PE FØR W LESS THAN 2100 M/SEC
1403  TE = TEIDAL
      RHØE= RHØINF *RE1
      KM =1.05
      PE =PSID
      GØ TØ 1405
1404  CØNTINUE
C     CALCULATE TE, RHØE,KM AND PE FØR W GREATER THAN 2100 M/SEC
      DØ = 4.016949 -1.49287E-2 *S
      D1 = -.895475  +4.51127E-3*S
      D2 = 9.28796E-2 -.54521E-3*S
      D3 = -4.746323E-3 +3.05088E-5 *S
      D4 = 11.611955E-5 -7.96241E-7 *S
      D5 = -10.86105E-7 +7.84415E-9 *S
      TE = TEIDAL*( ((((MØØ*D5+D4)*MØØ+D3)*MØØ+D2)*MØØ+D1)*MØØ +DØ )
```

```
      RERE = ((((((MØØ*(3.569058E-8 +3.77445E-9*S ) + (-.832101E-5 -
     16.34127E-7*S))*MØØ)+ (5.06022E-4 +3.2325E-5*S))*MØØ
     2+ (-1.334415E-2 -6.96885E-4* S))*MØØ +(.1975914+ 7.249941E-3
     3*S))*MØØ  + .269623- 2.47746E-2 * S
      RHØE= RHØINF* RE1*RERE
      HØ = .868 -.1325E-2 *S
      H1 = -3.98E-4 *S -.09081
      H2 = 1.2245E-4 * S  +.022684
      H3 = -1.0185E-5 *S  -1.637 E-3
      BU = W/1000. - 5.
      KM = ((  BU*H3 + H2)*.BU + H1)*BU + HØ
 1405 CØNTINUE
C     RØUTINE TØ CØMPUTE  V MELTING
20903 MUE= (SQRT(TE)*(1.50541E-7)/(1.+A19/TE))*GSEA
      MUW = (SQRT(TW) *(1.50541E-7))/(1.+A19/TW)*GSEA
      RINRE=RHØINF/RHØE
C     PRØBSTEIN  S  EQUATIØN FØR THE SHEAR STRESS
      TAWXØ=.727*(W/RFT*(1.-RINRE))**1.5 *(8./3.*RINRE)**.75
     1*(UMUE*TW)/(MUW*TE))**.447 *SQRT (2.*RHØE*MUW *TE/TW)
      IF (PVAP) 1809,1811,1809
 1811 TAWX=TAWXØ
      GØ TØ 1812
 1809 CØNTINUE
      TAWX=TAWXØ*(SI     *(1.+(SE1*SI)/(PS/PVAP-1.)))
 1812 CØNTINUE
      DØ 10017 IP=1,100
10017 VV(IP)=0.
      DØ 10018 IP=1,200
      ARG(IP)=0.
      TMU(IP)=0.
10018 ARG1(IP)=0.
      IDUM=0
      DYB= DY/DIV
      I4=4
      NK=(N-1)*MM+1
      DØ 105 J=1,NK
      AZK = J-1
      YBAR = DYB*AZK
      YB(J)= YBAR
      CALL LATLUM(IERR,IDUM,IØNE,N,I4,YBAR,Y,T,TT)
      IF (IERR) 101,102,101
  101 CØNTINUE
      WRITE (6,791)
  791 FØRMAT(1X50HERRØR IN INTERPØLATIØN PRØCEEDURE FØR SMALL T GRID)
      CALL DUMP(YBAR,YBAR,1,T(1),T(99),1,Y(1),Y(99),1)
C     ERRØR IN INTERPØLATIØN
      CØNTINUE
  102 CALL SUBMU (TT,B1,B2,B3,B4,MU,TAG )
      IF (TAG)104,103,104
  103 TMU(J)= 0.
      GØ TØ 106
  104 TMU(J)=MU
      IF (J-200) 105,103,103
  105 CØNTINUE
      JJ=NK
      GØ TØ 9918
  106 IF (J-1) 107,107,9919
 9919 CØNTINUE
      JJ=J-1
 9918 CØNTINUE
      YØ= YB(JJ)
```

```
      IF (JJ-4) 107,109,109
 107  DØ 108 JP=1,N
C     SET ARRAYS EQUAL ZERØ IF PRØFILE IS NØT LØNG ENØUGH
      TMU(JP)=0.
108   VV(JP) =0.
      VØØ=0.
      GØ TØ 116
 109  CØNTINUE
20902 CØNTINUE
      CØFFA = 2.*(2.*(PS-PØØ)/RFT**2 + RHØ*DWDT/RFT)
      CØFFB = 2.* TAWX
      DØ 20904 IL=1,JJ
20904 ARG(IL) =(CØFFA*YB(IL) + CØFFB) /TMU(IL)
      GØ TØ 10902
10902 CØNTINUE
      CALL SUBVI (ARG1,DYB,ARG,JJ)
 502  CØNTINUE
10903 CØNTINUE
      DVY=ARG1(JJ)
      DØ 2712 IK=1,JJ
 2712 ARG(IK) = ARG1(IK)-DVY
      CALL SUBVI(ARG1,DYB,ARG,JJ)
      VØØ = ARG1(JJ)
      JVK=JJ/MM+1
      DØ 2713 JZ=1,JVK
      JV =(JZ-1)*MM +1
      TMU(JZ) = TMU(JV)
 2713 VV(JZ)= ARG1(JV)
      IF (JV-JJ) 9796,9795,9795
C     SPECIAL CASE WHERE PRØFILE TERMINATED ØN EVEN GRID STEP
 9795 VV(JVK)=VØØ
 9796 CØNTINUE
      JZ=JVK
 504  CØNTINUE
      JX= N-JZ
      IF (JX)116,116,115
115   CØNTINUE
      DØ 1115 IZ=1,JX
      JVI = JZ +IZ
      TMU(JVI) =0.
 1115 VV(JVI)=VØØ
 116  CØNTINUE
      NN =N-1
      DØ 34 I=2,NN
      D2Y(I) =(T(I-1)-2.*T(I) +T(I+1))/DY2
      DTDY(I) =(T(I+1)- T(I-1))/(2.*DY)
      DTDTI(I) = RKCP*D2Y (I)
34    CØNTINUE
      DTDY(N)=0.
      D2Y (N)=0.
      DELT = ABSF(T(1)- TLAST(1))/4.
      IF (DELT-.5)35,36,36
35    DELT= .5
36    CØNTINUE
      TW= T(1)
      ITER=1
      GØ TØ (8021,8022),IRAD
 8022 CØNTINUE
      DØ 7098 NNN=1,N
      IF (T(NNN)-310.) 7099,7097,7097
 7097 CØNTINUE
```

```
 7098 CØNTINUE
      NNN=N
 7099 CØNTINUE
      IF (NNN-IYZ) 7096,7095,7095
 7096 IF (NNN-1) 7092,7092,7091
 7092 FØ=0.
      FYB=0.
      GØ TØ 7093
 7091 NNN= IYZ
 7095 CØNTINUE
      DØ 701 L=1,NNN
      DØ 700 J=1,NNN
 700  RG(J) = (T(J)    )**4 *EXP(-ALFR*(Y(L)+Y(J)))
      CALL CINTD (NNN,DY,RG,RG1)
      RA1(L) =RG1(NNN)
      NP1=NNN-L+1
      DØ 703 J=1,NNN
      MN1 =J-L+1
 703  RG(MN1)= (T(J)    )**4 *EXP(-ALFR*(Y(J) -Y(L)))
      CALL CINTD (NP1,DY,RG,RG1)
      ARG(L) = RG1(NP1)
      DØ 704 J=1,L
 704  RG(J)=(T(J)    )**4 *EXP(-ALFR*(Y(L)-Y(J)))
      CALL CINTD (L,DY,RG,RG1)
      RA (L) = RG1 (L)
 701  CØNTINUE
      DØ 706 I=1,NNN
 706  DFDY(I) = CØNST *(2.*T (I)**4 -ALFR * (RA(I)+ARG(I) +REFF
     1*RA1(I)))
      FØ = CØNST * ( REFF -1.) * RA1(1)
      FYB = CØNST * (RA(IYZ) - ARG(IYZ) + REFF * (RA1(IYZ)))
      NNN=NNN+1
 7093 DØ 7094 I=NNN,N
 7094 DFDY(I)=0.
 8021 CØNTINUE
 200  HE = W2/8374.88  +.24*TINF
      IF (TW) 8091,8091,8093
 8092 FØRMAT(1H11X32HTW NEGATIVE ØN ITERATIØN NUMBER I4)
 8091 WRITE (6,8092) ITER
      KEY =2
      GØ TØ 507
 8093 CØNTINUE
      HW=CØFHW(1)
      DØ 37 I=1,4
 37   HW=TW* HW + CØFHW(I+1)
      HEW=HE-HW
      QRAD= 1.378E-11 * E * TW**4
      QRADG = 0.
      IF (2100.-W)38,39,39
 38   WC= SQRT (A6/(A7+H))
      QCØNT = A8 *SQRT (RHØINF/RFT)* (W/WC)**A9* (HE-HW)/(HE-A10)
      GØ TØ 44
 39   CØNTINUE
      MUWAIR =SQRT (TW) * A18/(1.+A19/TW)
      KWAIR=CØFK(1)
      DØ 40 I=1,5
 40   KWAIR = TW*KWAIR+CØFK(I+1)
      TWTS=  TW/TS
      IF (1.6-TWTS) 41,41,42
 41   FNU = A16 +A17*TWTS
      GØ  TØ 43
```

```
 42    FNU =A14 + A15*TWTS
 43    CØNTINUE
       QCØNT = KWAIR/SQRT(MUWAIR)  *(TS-TW)*SQRT (TS*RS/TW)
      1* FNU* A13**.4 *SQRT (1.05*W/(2.*RFT))
44     CØNTINUE
       GØ TØ (460,460,45),KTR
 460   CØNTINUE
       QMØL = A11*RHØINF*W*(W2 - A12*(TW-TØØ))
       GØ TØ 46
 45    QAERØ = QCØNT
       GØ TØ 47
 46    QSLIP = (QMØL*QCØNT)/SQRT (QMØL**2 + QCØNT**2)
       IF (H-H70KM)619,619,622
619    GØ TØ (620,621,45),KTR
C      KTR= 1 WHEN H IS ABØVE H70 KM,KTR=2 DURING TRANSITIØN,
C      KTR= 3 AFTER TRANSITIØN
620    KTR =2
       TR2  = TIME + 10.*DT
       TR1 = TIME
       GØ TØ 622
621    IF (TR2- TIME) 624,624,623
624    KTR=3
       GØ TØ 45
623    QAERØ =QSLIP + (QCØNT-QSLIP)*(TIME-TR1)/(TR2-TR1)
       GØ TØ 47
622    CØNTINUE
       QAERØ =Q SLIP
 47    CØNTINUE
       T1= 1./TW
       PVAP =A1* EXP ((A2*T1 +A3)*T1 +A4)
       PVAPS=PVAP
       PVAP=PVAP**M1 *PS**(1.-M1)
       IF  (PVAP-PS) 1515,1516,1516
C      CØRRECT GUESS FØR TW
 1516  TW=(TW-TLAST(1))* .5 +TLAST(1)
       T(1) = TW
       GØ TØ 200
 1515  CØNTINUE
       IF (ABSF(PVAP) - 1.E-20)  48,48,49
 48    SI=1.
       PVAP=0.
       CWEQ=0.
       VI=0.
       GØ TØ 52
 49    CWEQ = 1./(1.+ M/MVAP * (PS/PVAP- 1.))
       SI  =(1.-CWEQ )/(1.-CWEQ*(1.- C68*(M/MVAP)**C26))
       IF (ABS(SI-1.) -1.E-7) 48,48,9797
 9797  CØNTINUE
       IF (ALFV) 51,50,51
 50    VW = -1./RHØ*SI *QAERØ/HEW * CWEQ/(1.-CWEQ)
       GØ TØ 52
 51    AVM =SQRT   (2.*PI *R * MVAP * TW )/(ALFV*((PS-PVAP)*M
      1+ PVAP*MVAP) )
       VI=1./(2.*AVM*RHØ)*(1.-CWEQ+AVM* SI*QAERØ/HEW -
      1SQRT ((1.-CWEQ+AVM*SI*QAERØ/HEW)** 2 +4.*AVM*CWEQ*SI
      2*QAERØ/HEW))
 52    CØNTINUE
       Q2= QAERØ * SI                   + RHØ*VI*HV
       GØ TØ  (8077,8088),IRAD
 8077  Q2=Q2-QRAD
 8088  CØNTINUE
```

```
8111  Q22=-Q2
8113  DTDY(1)=Q22/K
      DØ 211 IQ=1,N
 211  DTDT(IQ) =DTDTI(IQ) -DTDY(IQ)*(VV(IQ)+VW)
     1-1./(RHØ*CP)*DFDY(IQ)
      T(2) = .33333333 * (TW-T(4)) + T (3)
      IF (T(2) -TØ) 1560,1561,1561
 1560 T(2)=TØ
 1561 CØNTINUE
      DTDT(N)=0.
      DØ 53  I=1,2
 53   DTDT(I) = (T(I) -TLAST(I))/DT
      DTDY(2) =(T(3) -T(1))/(2.*DY)
      DTDY(3) =  (T(4)-T(2))/(2.*DY)
      D2Y(1)=1./K*(RHØ*CP*(DTDT(1)+VI*DTDY(1))+DFDY(1))
      D2Y(2)=1./K*(RHØ*CP*(DTDT(2)+(VI+VV(2))*DTDY(2))
     1+DFDY(2))
      D2Y(3) =  (T(4) -2.*T(3) +T(2))/DY2
      DTDT(3) =RKCP*D2Y(3)-DTDY(3)*(VV(3)+VW)
     1-1./(RHØ*CP)*DFDY(3)
      DØ 54 I=1,IYZ
54    ARG(I)=DTDT(I)+DTDY(I)*  (VV(I)+VW)
      CALL CINT (ARG,DY,FI,IYZ)
      Q1 =  -K *DTDY (IYZ) + RHØ*CP * FI
      Q1=Q1+FYB-FØ
 556  CØNTINUE
 506  CØNTINUE
      IF(ABS(Q2)-QTØL) 65,65,579
C     IF ABS(Q2) IS LESS THAN QTØL, THE ITERATIØN IS SKIPPED
 579  CØNTINUE
      TØL=ABS(Q2*Q2TØL)
      D=Q2-Q1
      IF(ABSF(D) - TØL )65,65,55
 55   IF (ITER-2) 56,60,60
 56   E2=D
      TW2=TW
      IF (D) 57,57,58
 57   TW =TW-DELT
      GØ TØ 61
 58   TW= TW+DELT
 59   GØ TØ 61
 60   E1= E2
      TW1=TW2
      E2= D
      TW2 =TW
      IF (ABSF( E2-E1)-.000001)65,62,62
 62   TW= (TW1*E2 - TW2*E1)/(E2-E1)
      ATS= TW-TW2
      IF (ABS(ATS)-100.) 9771,9772,9772
C     LIMIT  THE CHANGE IN TEMPERATURE TØ 100 DEGREES MAX.
 9772 TW= TW2 + 100.*ABS(ATS)/ATS
 9771 CØNTINUE
      IF (ITER-15) 61,61,1661
 1661 CØNTINUE
712   FØRMAT(46HITERATIØN FØR TW FAILED TØ CØNVERGE,END ØF RUN)
      WRITE (6,712)
      KEY =2
      GØ TØ 507
 61   ITER=ITER+1
      T(1)=TW
      CØNTINUE
```

```
      GØ TØ 200
 65   CØNTINUE
      T(1)=TW
C     ITERATIØN CØNVERGED FØR TW
      VØØ=VØØ+VW
      DØ 650 I=1,N
 650  VV(I)=VV(I) +VW
C     SET UP TABLES ØF VOO AND VW TØ INTEGRATE ØVER TIME.
      TABVI(3)=TABVI(2)
      TABVI(2)=TABVI(1)
      TABVØ(3)=TABVØ(2)
      TABVØ(2)=TABVØ(1)
      TABVI(1) = VW
      TABVØ(1) = VØØ
      IF (IKI-2) 752,653,654
 752  YS=YS-DT*TABVØ(1)
      YSW=YSW+DT*TABVI(1)
      GØ TØ 655
 653  YS=YS-DT2*(TABVØ(1)+TABVØ(2))
      YSW=YSW+DT2*(TABVI(1)+TABVI(2))
      GØ TØ 655
 654  YS=YS-DT/12.*(-TABVØ(3)+8.*TABVØ(2)+5.*TABVØ(1))
      YSW=YSW+DT/12.*(-TABVI(3)+8.*TABVI(2)+5.*TABVI(1))
 655  IKI=IKI+1
      IF (ABS(YS)-1.E-6) 66,66,9707
 9707 CØNTINUE
      GØ TØ (161,162),METHØD
 161  CØNTINUE
      RFØYS = RFØ-YS
      IF (RFØYS) 6796,6796,16111
 6796 CØNTINUE
      IF (SPHERE) 6798,67,6798
 6798 CØNTINUE
      WRITE (6,6797)
 6797 FØRMAT(1X25H HALF ØF BØDY BURNED AWAY)
      KEY=2
      GØ TØ 507
16111 RFT = RFØ + .5 * YS**2/RFØYS
      MASS = MØ -RHØ*PI*YS*(RFØ*(RFØ-YS*.5)+YS**2/6.)
      RFV=RFT
      GØ TØ 163
 162  CØNTINUE
      RFT = RFØ*(1.+.5* (1.-EXP (-10.*YS/RFØ))-.32*(YS/RFØ)**2)
      LH = RFT -RFØ +YS
      Z1 =(RFØ**2 - RFT**2 + LH**2)/(2.*LH)
      ETA1 =        (RFØ**2 -Z1**2)
      IF (ETA1) 10904,6641,6641
10904 CØNTINUE
      ETA1=RFØ**2
      GØ TØ 6641
 6641 ETA1=SQRT(ETA1)
      FC1 = RFØ + YSW
      RFV=.5*(Z1+FC1) + ETA1**2/(2.*(Z1+FC1))
      FJS =RFV - RFØ -YSW
      MASS =MØ -RHØ*PI*(RFØ**2 *(Z1+RFØ)-1./3.* (Z1**3+RFØ**3)
     1-RFV**2*(Z1+FC1) +1./3.*((Z1-FJS)**3 + (FC1+FJS)**3) )
 163  CØNTINUE
      IF  (THICK-ABSF(YS))67,67,66
 713  FØRMAT(28H1BØDY BURNED AWAY,END ØF RUN)
 67   WRITE (6,713)
      KEY =2
```

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      GØ TØ 507
 66   CØNTINUE
      IF (KPRINT-IPRINT) 508,7705,7705
 7705 CØNTINUE
      KEY=1
 507  CØNTINUE
C     KEY = 1 IF REGULAR PRINT, IF TERMINAL PRINT KEY =2
      PHIT= 180./PI * ATAN2(V,U)
      YSN=-YSW
      GØ TØ (8023,8024),IRAD
 8024 CØNTINUE
 1711 FØRMAT(7H1TIME= E16.8,/8HOH        E16.8,8H WINF     E16.8,8H MACH     E
     116.8,8H PHI      E16.8,8H DWDT     E16.8,/8H UINF     E16.8,8H VINF     E1
     26.8,8H DU/DT    E16.8,8H DV/DT    E16.8,8H CD       E16.8,/8H RFT      E16
     3.8,8H MASS     E16.8,8H YS       E16.8,8H YSW      E16.8,8H RINF     E16.8
     4,/8H TINF     E16.8,8H PINF     E16.8,8H AINF     E16.8,8H MUINF    E16.8,
     58H GINF     E16.8,/8H RE       E16.8,8H RHØE     E16.8,8H TE       E16.8,8
     6H PE       E16.8,8H MUE      E16.8,/8H KM       E16.8,8H M        E16.8,8H
     7 HE       E16.8,8H HW       E16.8,8H PVAPS    E16.8,/8H PVAP     E16.8,8H
     8AVM      E16.8,8H QARØ     E16.8,8H SI       E16.8,8H F(Ø)     E16.8,/8H T
     9AW      E16.8,8H INT      E16.8,8H F(YB)    E16.8,///)
      WRITE (6,1711) TIME,H,W,MØØ,PHIT,DWDT,U,V,DUDT,DVDT,CD,RFT,MASS,
     1YS,YSN,RHØINF,TØØ,PØØ,AØØ,MUINF,GT,RE,RHØE,TE,PE,MUE,KM,M,HE,HW,
     2PVAPS,PVAP,AVM,QAERØ,SI,FØ   ,TAWX,FI,FYB
 1712 FØRMAT(4H Y= E13.6,4H T= E14.6,5H TP= E14.6,6H TP2= E14.6,5H DF= E
     116.8,4H V= E13.6,7H DTDT= E14.6)
      J=1
      DØ   9721    I=1,N
      J=J+1
 1713 WRITE (6,1712) Y(I),T(I),DTDY(I),D2Y(I),DFDY(I),VV(I),DTDT(I)
      IF (IYZ-I) 9736,9738,9738
 9736 IF (T(I)-301.)9737,9738,9738
 9738 CØNTINUE
      IF (J-45)9722,9723,9723
 9723 WRITE  (6,9713)
      J=0
 9722 CØNTINUE
 9721 CØNTINUE
 9737 CØNTINUE
      GØ TØ 8025
 8023 CØNTINUE
 8031 FØRMAT(7H1TIME= E16.8,/8HOH        E16.8,8H WINF     E16.8,8H MACH     E
     116.8,8H PHI      E16.8,8H DWDT     E16.8,/8H UINF     E16.8,8H VINF     E1
     26.8,8H DU/DT    E16.8,8H DV/DT    E16.8,8H CD       E16.8,/8H RFT      E16
     3.8,8H MASS     E16.8,8H YS       E16.8,8H YSW      E16.8,8H RINF     E16.8
     4,/8H TINF     E16.8,8H PINF     E16.8,8H AINF     E16.8,8H MUINF    E16.8,
     58H GINF     E16.8,/8H RE       E16.8,8H RHØE     E16.8,8H TE       E16.8,8
     6H PE       E16.8,8H MUE      E16.8,/8H KM       E16.8,8H M        E16.8,8H
     7 HE       E16.8,8H HW       E16.8,8H PVAPS    E16.8,/8H PVAP     E16.8,8H
     8AVM      E16.8,8H QARØ     E16.8,8H SI       E16.8,8H QRAD     E16.8,/8H T
     9AW      E16.8,8H INT      E16.8,///)
      WRITE (6,8031) TIME,H,W,MØØ,PHIT,DWDT,U,V,DUDT,DVDT,CD,RFT,MASS,
     1YS,YSN,RHØINF,TØØ,PØØ,AØØ,MUINF,GT,RE,RHØE,TE,PE,MUE,KM,M,HE,HW,
     2PVAPS,PVAP,AVM,QAERØ,SI,QRAD,TAWX,FI
 8032 FØRMAT(4H Y= E13.6,4H T= E14.6,5H TP= E14.6,6H TP2= E14.6,5H MU= E
     114.7,4H V= E13.6,7H DTDT= E14.6)
      J=1
      DØ 9711  I=1,N
      J=J+1
 8033 WRITE (6,8032) Y(I),T(I),DTDY(I),D2Y(I),TMU(I),VV(I),DTDT(I)
      IF (IYZ-I) 9726,9728,9728
```

```
 9726 IF (T(I)-301.) 9727,9728,9728
 9728 CØNTINUE
      IF (J-45) 9714,9712,9712
 9712 WRITE (6,9713)
      J=0
 9713 FØRMAT(1H1)
 9714 CØNTINUE
9711  CØNTINUE
 9727 CØNTINUE
 8025 CØNTINUE
      IF (KEY-2)  9781,9782,9782
 9783 FØRMAT(1H1,
     11H 6HZETA1 E16.8,6H ETA1 E16.8,6H  H   E16.8,6H RFV  E16.8,6H  J
     1 E16.8)
 9782 WRITE (6,9783) Z1,ETA1,LH,RFV,FJS
 9781 CØNTINUE
C     THIS PARTS CØMPUTES ELAPSED TIME ØF CASE AND PRINTS
      GØ TØ (8121,8122,8121),KEY
 8122 CØNTINUE
      CALL SCLØCK(DATE,CTIME,ESEC,E6ØSEC)
      IF (DATE)8123,8121,8123
 8123 CØNTINUE
      TØTTIM=E60SEC-SEC
      ISEC1 =  TØT TIM
      ISEC =  ISEC1/60
      ISEC2 = ISEC * 60
      ISEC60 =   ISEC1 - ISEC 2
      ISEC3 = ISEC/60
      ISEC4 = ISEC 3*60
      ISEC5=ISEC-ISEC4
 8124 FØRMAT(31H0ELAPSED TIME ØN THIS CASE WAS I4,9H MINUTES,I4,5H AND
     1I4,12H 60TH SECØND)
      WRITE(6,8124) ISEC3, ISEC5,ISEC60
 8121 CØNTINUE
C     END ØF ELAPSED TIME PART
      GØ TØ (508,1,506),KEY
 508  CØNTINUE
      TMM = TIME - MXTIM
      IF (TMM)  5049,696,696
 5049 IF (ABS(TMM)-DT/4.) 696,697,697
 696  CØNTINUE
      KEY= 2
4902  FØRMAT(51H0TRAJECTØRY TERMINATED BECAUSE MAXIMUM TIME REACHED)
      WRITE (6,4902)
      GØ TØ 507
 697  CØNTINUE
      TM= TIME
      UN =U
      VN =V
      HN=H
      DØ 666 I=1,N
 666  TLAST(I)=T(I)
      IF (KPRINT-IPRINT) 515,516,516
 516  KPRINT=0
 515  CØNTINUE
      GØ TØ 1000
500   CØNTINUE
C     RØUTINE  TØ CØMPUTE THE TRAJECTØRY EQUATIØNS
      H=HN+V*(TIME-TM)
      IF (H-HMAX) 4917,4915,4915
 4915 WRITE (6,4914 )
```

```
C     CASE IS TERMINATED IF ALTITUDE EXCEEDS HMAX
C     THIS IS NECESSARY TØ PRØTECT AGAINST A BØUNCING BØDY  THAT
C     DØES NØT  RE-ENTER  THE EARTHS ATMØSPHERE.
 4914 FØRMAT(1H130HBØUNCING BØDY, JØB TERMINATED.)
      KEY=2
      GØ TØ 507
 4917 CØNTINUE
      IF (H) 4900,4900,4907
4900  KEY=2
4901  FØRMAT(51H0TRAJECTØRY TERMINATED BECAUSE ØF IMPACT WITH EARTH)
      WRITE (6,4901)
      GØ TØ 507
 4907 CØNTINUE
C     USE ALTITUDE RØUTINE TØ ØBTAIN  FUNCTIØNS ØF  ALTITUDE
      PR(1) =H
      ERR=0
      CALL PRA63(PR,ERR)
      IF (ERR -1.) 1107,1108,1107
 1108 WRITE(6,1109)
 1109 FØRMAT(45H0ERRØR ALTITUDE FUNCTIØN RØUTINE, END ØF JØB.)
 1107 CØNTINUE
      PØØ = PR(2)*10000.
      TØØ = PR(3)
      RHØINF = PR(6)
      MUINF =  PR(7)
      AØØ =  PR(9)
      M   =  PR(10)
      IF (M) 1859,1860,1859
 1860 M=28.9644
 1859 CØNTINUE
      GT=GSEA/((H +RSEA)/RSEA)**2
      W2= U**2 +V**2
      W = SQRT (W2)
      MØØ = W/AØØ
      RE  =(RHØINF*W *2.* RFØ)/MUINF
C     CALCULATE  DRAG
      IF (H-HKM) 14,14,15
15    IF (MØØ-9.)16,16 ,17
17    CD=2.
      GØ TØ 30
16    CD =1.68 + 2.85/MØØ
      GØ TØ 30
C     IF SPHERE IS ZERØ,TEKTITE IS HEMISPHERICAL,ØTHERWISE IT IS SPHERICAL
14    IF (SPHERE)19,26,19
19    IF (MØØ-2.)21,21,20
20    CD = .9 +MØØ/SQRT (RE)
      GØ TØ 30
21    IF (MØØ- 1.26)23,23,22
22    CD = 1.034 -.027*MØØ
      GØ TØ 30
23    IF (MØØ-.8)24,24,25
 24   CD =.5
      GØ TØ 30
25    CD= .812*MØØ -.023
      GØ TØ 30
  26  IF (MØØ-2.) 28,28,27
  27  CD=1.35 + MØØ/SQRT (RE)
      GØ TØ 30
  28  CD= 1.35
  30  CØNTINUE
      TERM2 =((.5*RHOINF*CD*W2 +.75*RHØ*VW**2)*PI*RFØ**2)/(W*MASS)
```

```
      DUDT = -U*V/(H+RSEA) -U*TERM2
      DVDT = -GT + U**2/(      (H+RSEA)) -V*TERM2
31    CØNTINUE
C     END ØF RØUTINE TØ CØMPUTE D.E FØR TRAJECTØRY
 511  CØNTINUE
      GØ TØ (1000,1001,1002,1003,1004),KR
      END
```

```
      SUBROUTINE CINTD (N,DEL,GAM,T)
      DIMENSION GAM (100),T(100)
      IF(N-1) 81,81,82
   81 T(1)=0.
      RETURN
   82 IF(N-2) 83,83,84
   83 T(1)=0.
      T(2)=(GAM(1)+GAM(2))*DEL/2.
      RETURN
   84 IF(N-3) 85,85,86
   85 T(1)=0.
      T(I)=(GAM(1)+GAM(2))*DEL/2.
      T(3)=(GAM(1)+4.*GAM(2)+GAM(3))*DEL/3.
      RETURN
   86 IF(N-4) 87,87,88
   87 T(1)=0.
      T(2)=(GAM(1)+GAM(2))*DEL/2.
      T(3)=(GAM(1)+4.*GAM(2)+GAM(3))*DEL/3.
      T(4)=3.*(GAM(1)+3.*GAM(2)+3.*GAM(3)+GAM(4))*DEL/8.
      RETURN
   88 T(1)=0.
      T(2)=DEL*(.348611111*GAM(1)+.897222222*GAM(2)-.366666667*GAM(3)+.1
     147222222*GAM(4)-.026388889*GAM(5))
      T(3)=DEL*(.322222222*GAM(1)+1.377777778*GAM(2)+.266666667*GAM(3)+.
     1044444444*GAM(4)-.011111111*GAM(5))
      J=N-1
      DØ 80 I=4,J
80    T(I)=T(I-1)+DEL*(.015277778*GAM(I-3)-.102777778*GAM(I-2)+.63333333
     13*GAM(I-1)+.480555556*GAM(I)-.026388889*GAM(I+1))
      T(J+1)=T(J)+DEL*(-.026388889*GAM(J-3)+.147222222*GAM(J-2)-.3666666
     167*GAM(J-1)+.897222222*GAM(J)+.348611111*GAM(J+1))
      RETURN
      END
```

```
      SUBRØUTINE CINT(ARG,H,FI,N)
      DIMENSIØN ARG(100)
      M=N-1
      H24=H/24.
      DØ 1919 I=1,M
      IF (I-1) 1,1,3
1     FI=  H24*( 9.*ARG(1)+19.*ARG(2) -5.*ARG(3) +ARG(4))
      GØ TØ 5
2     FI=FI +H24*(-ARG(I-1) +13.*(ARG(I)+ARG(I+1)) -ARG(I+2))
      GØ TØ 5
3     IF (I-M) 2,4,4
4     FI=FI + H24*(ARG(I-2)-5.*ARG(I-1) +19.*ARG(I) + 9.*ARG(I+1))
5     CØNTINUE
 1919 CØNTINUE
      RETURN
      END
C     SUBRØUTINE TØ CØMPUTE VISCØSITY MU,TEMP IS TEMPERATURE,B1,B2,B3
C     ARE CØFFICIENTS,TAG RETURNS AS ZERØ IF MU REAL LARGE,ØTHERWISE 1
```

```
      SUBRØUTINE SUBMU(TEMP,B1,B2,B3,B4,FU,TAG )
      A=B2/(TEMP-B3) +B4
      IF (A-50.)2,1,1
1     TAG=0.
      GØ TØ 3
2     FU=B1*EXP(A)
      TAG=1.
3     CØNTINUE
      RETURN
C     END ØF VISCØSITY SUBRØUTINE
      END
```

```
      SUBRØUTINE  SUBVI(FI,H,Y,N )
      DIMENSIØN FI(200),Y(200)
      FI(1)=0.
      H24 =H/24.
      IF (N-4 ) 1,2,2
1     GØ TØ 8
2     DØ 7  I=2,N
      IF (I-2) 3,3,4
3     FI(I) =H24*(9.*Y(I-1) +19.* Y(I)-5.*Y(I+1) +Y(I+2))
      GØ TØ 7
4     IF (I-N) 5,6,5
5     FI(I) =FI(I-1) +H24* (-Y(I-2) +13.*(Y(I-1) +Y(I)) -Y(I+1))
      GØ TØ 7
6     FI(I) =FI(I-1) +H24*(Y(I-3)-5.*Y(I-2)+19.*Y(I-1)+9.*Y(I))
7     CØNTINUE
      RETURN
8     DØ 9 I=1,N
9     FI(I) =0.
      RETURN
      END
```

```
C  LAGRANGE TABLE LØØK UP AND MULTIPLE INTERPØLATIØN
C  BY  TØMMY J. HEINTSCHEL
C  GENERAL ELECTRIC CØMPANY , FLIGHT ANALYSIS UNIT
C  FØRTRAN IV LANGUAGE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      SUBRØUTINE LATLUM(IERR,IDUM,M,N,P,ARG,X1,Y1,ANS1)
C     IERR = ERRØR SWITCH
C            IF ERRØR ØCCURS , IERR = NØNZERØ
C            IF NØ ERRØR ØCCURS , IERR = 0
C     IDUM = PRESENT TABLE LØCATIØN USED BY SUBRØUTINE.
C            BEFØRE ENTERING SUBRØUTINE FIRST TIME , PRØGRAMMER
C                MUST SET IDUM=0
C     M    =  NUMBER ØF DEPENDENT TABLES
C     N    =  NUMBER ØF TABULAR PØINTS PER TABLE
C     P    =  NUMBER ØF PØINTS USED FØR EACH INTERPØLATIØN
C             P EQUAL TØ ØR LESS THAN 10
C     ARG  =  LØCATIØN ØF ARGUMENT (X)
C     X1   =  LØCATIØN ØF FIRST VALUE ØF INDEPENDENT TABLE
C     Y1   =  LØCATIØN ØF FIRST VALUE ØF DEPENDENT TABLES
C     ANS1 = LØCATIØN AT WHICH THE FIRST ANSWER IS TØ BE STØRED
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSIØN X1(N),Y1(N,M),ANS1(M)
      INTEGER P
      IF (P .GT. 10) GØ TØ 25
      IERR=0
      IF (IDUM .EQ. 0) IDUM=1
      DØ 10 J=IDUM,N
      IF (ARG .GT. X1(J)) GØ TØ 10
      GØ TØ 20
   10 CØNTINUE
      IF (ARG .EQ. X1(J)) GØ TØ 50
   25 IERR=99999
      RETURN
   28 J=J-1
   50 DØ 40 K=1,M
   40 ANS1(K)=Y1(J,K)
      RETURN
   20 IF (ARG .EQ. X1(J)) GØ TØ 50
      J=J-1
      IF (J .LT. 1) GØ TØ 25
      IF (ARG .LE. X1(J)) GØ TØ 20
      J=J+1
      IDUM=J
      DØNA1=2.*(X1(J)-ARG)
      DØNA2=2.*(ARG-X1(J-1))
      DØNA3=ABS(X1(J))+ABS(X1(J-1))
      IF (ABS(DØNA1/DØNA3) .LE. .0000008) GØ TØ 50
      IF (ABS(DØNA2/DØNA3) .LE. .0000008) GØ TØ 28
      IF (P-(P/2)*2 .EQ. 0) GØ TØ 80
      IF (ABS(DØNA1) .LE. ABS(DØNA2)) GØ TØ 80
      ISTRT= J - (P+1)/2
      GØ TØ 70
   80 ISTRT= J -  P/2
   70 ISTØP=ISTRT+P-1
      IF (ISTRT .GE. 1 ) GØ TØ 90
      ISTRT= 1
      ISTØP= P
      GØ TØ 100
   90 IF (ISTØP .LE. N) GØ TØ 100
```

```
      ISTRT= N-P+1
      ISTØP= N
  100 DØ 120 K=1,M
      ANS1(K)=0.
      PPRØD=1.
      DØ 130 L=ISTRT,ISTØP
      PPRØD=PPRØD*(ARG-X1(L))
      PRØD=1.
      DØ 140 LL=ISTRT,ISTØP
      IF (L .EQ. LL ) GØ TØ 140
      PRØD=PRØD*(X1(L)-X1(LL))
  140 CØNTINUE
      ANS1(K)=ANS1(K)+(Y1(L,K)/PRØD)/(ARG-X1(L))
  130 CØNTINUE
  120 ANS1(K)=ANS1(K)*PPRØD
      RETURN
      END
```

```
      SUBRØUTINE PRA63(PR,ERRØR)
      DIMENSIØN PR(15),PB(14),ZI(5),PK(6,5),RHØK(6,3),TK(6,5),VTK(6,3),
     1ZB(14),TMB(14),LMB(14),DMB(14),TB(14),MB(14)
      REAL LMB,MB,MWT
C ENTER WITH PR(1)=CURRENT ALTITUDE
C CALCULATED TABLE AT RETURN
C PR(1)=CURRENT ALTITUDE        Z
C PR(2)=PRESSURE                PRES
C PR(3)=KINETIC TEMPERATURE     TEMPK
C PR(4)=VIRTUAL TEMPERATURE     TEMPV   (=0.0 BEYØND 90,000.0 METERS)
C PR(5)=MØLECULAR TEMPERATURE   TEMPM
C PR(6)=DENSITY                 DENS
C PR(7)=VISCØSITY               VISCØS
C PR(8)=KINEMATIC VISCØSITY     VISK    (=0.0 BEYØND 90,000.0 METERS)
C PR(9)=SPEED ØF SØUND          SPDSØ
C PR(10)=MØLECULAR WEIGHT       MWT
C PR(11)=SEA LEVEL PRESSURE     PSL
C PR(12)=PRESSURE RATIØ         PRAT
C PR(13)=DENSITY RATIØ          DR
C PR(14)=VISCØSITY RATIØ        VR
C PR(15)= PRESSURE DIFFERENCE   DELP
      ERRØR=0.
    1 Z=PR(1)
      IF(Z.GE.0..AND.Z.LE.700000.) GØ TØ 20
      ERRØR=1.
      IF(Z.LT.0.) Z=0.
      IF(Z.GT.700000.) Z=700000.
   20 N=1
      IF( Z -83004.) 40,30,30
   40 IF( Z -ZI(N))60,50,50
   50 N=N+1
      GØ TØ 40
   60 Z2   =Z*Z
      Z3   =Z2*Z
      Z4   =Z2*Z2
      Z5   =Z2*Z3
      GØ TØ 100
   30 IF(Z-90000.) 300,70,70
   70 IF(Z-ZB(N))85,400,80
   80 N=N+1
      GØ TØ 70
   85 N=N-1
      GØ TØ 400
C***** KINETIC TEMPERATURE FØR (0 TØ 83004)
  100 TEMPK=TK(1,N)+TK(2,N)*Z+TK(3,N)*Z2  +TK(4,N)*Z3  +TK(5,N)*Z4  +TK(
     16,N)*Z5
      IF(Z-28000.) 120,140,140
C***** PRESSURE FØR (0 TØ 28000)
  120 PRES= 10.0000000*EXP(PK(1,N)+PK(2,N)*Z+PK(3,N)*Z2+PK(4,N)*Z3+PK(5
     1,N)*Z4+PK(6,N)*Z5)
C***** DENSITY FØR (0 TØ 28000)
      DENS=(1.16790729)*EXP(RHØK(1,N)+RHØK(2,N)*Z+RHØK(3,N)*Z2  +RHØK(4,
     1N)*Z3+RHØK(5,N)*Z4+RHØK(6,N)*Z5)
      IF(Z-10832.1)130,130,160
C***** VIRTUAL TEMPERATURE FØR (0 TØ 12000)
  130 TEMPV=(VTK(1,N)+VTK(2,N)*Z+VTK(3,N)*Z2  +VTK(4,N)*Z3  +VTK(5,N)*Z4
     1  +VTK(6,N)*Z5)
      GØ TØ 170
C***** PRESSURE FØR (28000 TØ 83004)
  140 PRES=.000980665*   EXP(PK(1,N)+PK(2,N)*Z+PK(3,N)*Z2  + PK(4,N)*Z3+
     1  PK(5,N)*Z4  +PK(6,N)*Z5)
```

```
C*****  DENSITY FØR (28000 TØ 90000)
  150 DENS=34.83676*(PRES/TEMPK)
  160 TEMPV=TEMPK
C***** VISCØSITY (0 TØ 90000)
  170 VISCØS=(.000001458*SQRT(TEMPK*TEMPK*TEMPK))/(TEMPK+110.4)
C***** KINEMATIC VISCØSITY FØR (0 TØ 90000)
      VISK=VISCØS/DENS
C***** SPEED ØF SØUND (0 TØ 90000)
      SPDSØ=20.0468*SQRT(TEMPV)
      MWT=28.9644
      TEMPM=TEMPK
C***** VISCØSITY RATIØ  (0 TØ 700000)
  180 VR=VISCØS/.00001830243
C***** PRESSURE RATIØ (0 TØ 700000)
      PRAT=PRES/10.1701472
C***** DENSITY RATIØ FØR (0 TØ 700000)
      DR=DENS/1.18354674
C*****  PRESSURE DIFFERENCE FØR (0 TØ 700000)
      DELP=PSL-PRES
C***CALCULATIØNS CØMPLETE , RETURN TØ MAIN PRØGRAM
      GØ TØ 500
  300 TEMPK=180.65
C***** PRESSURE FØR (83004 TØ 90000)
      PRES=PBASE*EXP((-1.373301523E12*( Z -83004.))/(180.65*(6344860.+Z
     1) *(6344860.+83004.)))
      GØ TØ 150
C***** MØLECULAR WEIGHT FØR (90000 TØ 700000)
  400 MWT=MB(N)+DMB(N)*( Z -ZB(N))
C***** MØLECULAR TEMPERATURE FØR (90000 TØ 700000)
      TEMPM=TMB(N)+LMB(N)*(Z-ZB(N))
C***** KINETIC TEMPERATURE FØR (90000 TØ 700000)
      TEMPK=(MWT/28.9644)*TEMPM
      PRES=EXP(ALØG(PB(N))+(1.373301523E12/(LMB(N)*(6344860.+Z)*(6344860
     1.+ZB(N))))*ALØG(TMB(N)/(TMB(N)+(LMB(N)*( Z -ZB(N))))))
      DENS=  34.83676*PRES/TEMPM
      VISCØS=(.000001458*SQRT(TEMPM*TEMPM*TEMPM))/(TEMPM+110.4)
      VISK=0.
      VR=VISCØS/.00001830243
      SPDSØ=20.0468*SQRT(TEMPM)
      TEMPV=TEMPK
      GØ TØ 180
500   PR(2)=PRES
      PR(3)=TEMPK
      PR(4)=TEMPV
      PR(5)=TEMPM
      PR(6)=DENS
      PR(7)=VISCØS
      PR(8)=VISK
      PR(9)=SPDSØ
      PR(10)=MWT
      PR(11) = PSL
      PR(12)=PRAT
      PR(13)=DR
      PR(14)=VR
      PR(15)=DELP
      RETURN
      DATA PSL /10.1701472/
      DATA PBASE/6.23101759E-5/
      DATA (ZI(I),I=1,5)/10832.1,17853.3,28000.,49000.,83004./
      DATA ((PK(I,J),I=1,6),J=1,5)/1.6871582E-2,-1.1425176E-4,-1.3612327
     XE-9,7.3624145E-14,-1.0800315E-17,3.3046432E-22,-7.9910777E-2,-8.10
```

```
X46438E-5,-5.5522383E-9,3.1116969E-13,-1.6687827E-17,3.8319351E-22,IX401230
X9.8414277E-1,-2.6976917E-4,8.5227541E-9,-3.9620263E-13,1.0146471E-IX401240
X17,-1.0264318E-22,                                                IX401250
X1.14118495E1,-4.11497477E-4,1.33664855E-8,-3.59518975E-13,        IX401260
X5.10097254E-18,-2.89055894E-23,                                   IX401270
X9.99324461,-2.58298177E-4,3.76139346E-9,-4.20887236E-14,          IX401280
X1.60182148E-19,-1.92508927E-25/                                   IX401290
 DATA ((RHØK(I,J),I=1,6),J=1,3)/1.3302117E-2,-8.8502064E-5,-4.21430IX401300
X56E-9,5.9517557E-13,-3.9744789E-17,7.8771273E-22,1.2667122E-1,    IX401310
X-1.3373147E-4,2.0667371E-9,2.3396109E-13,-3.2562503E-17,7.9035209EIX401320
X-22,9.2751266E-1,-1.4349679E-4,-2.8271736E-9,4.7480092E-14,       IX401330
X1.8863246E-18,-4.2702411E-23/                                     IX401340
 DATA ((TK(I,J),I=1,6),J=1,5)/2.9667877E2,-6.7731001E-3,8.4619805E-IX401350
X7,-1.7004049E-10,1.1451454E-14,-2.4898788E-19,                    IX401360
X2.6892151E2,4.3075352E-3,-8.9159672E-7,-2.8929791E-11,5.0724856E-1IX401370
X5,-1.1490372E-19,                                                 IX401380
X3.7064557E2,-3.2858965E-2,2.0645636E-6,-4.3283944E-11,-5.7507242E-IX401390
X17,8.2924583E-21,                                                 IX401400
X2.044798E1,2.07698384E-2,-8.63038789E-7,1.66392417E-11,           IX401410
X-9.30076185E-17,-4.09005108E-22,                                  IX401420
X-4.98865953E2,3.92137281E-2,-4.95180601E-7,-3.26219854E-12,       IX401430
X 9.66650364E-17,-4.78844279E-22/                                  IX401440
 DATA ((VTK(I,J),I=1,6),J=1,3)/2.9937265E2,-7.717628E-3,9.4867202E-IX401450
X7,-1.7136592E-10,1.1074297E-14,-2.3294094E-19,                    IX401460
X2.6892151E2,4.3075352E-3,-8.9159672E-7,-2.8929791E-11,5.0724856E-1IX401470
X5,-1.1490372E-19,                                                 IX401480
X3.7064557E2,-3.2858965E-2,2.0645636E-6,-4.3283944E-11,-5.7507242E-IX401490
X17,8.2924583E-21/                                                 IX401500
 DATA (ZB(I),I=1,14)/                                              IX401510
X9.E4,1.E5,1.1E5,1.2E5,1.5E5,1.6E5,1.7E5,1.9E5,2.3E5,3.E5,4.E5,5.E5IX401520
X,6.E5,7.E5/                                                       IX401530
 DATA (TMB(I),I=1,14)/          180.65,210.65,260.65,360.65,960.65, IX401540
X1110.65,1210.65,1350.65,1550.65,1830.65,2160.65,2420.65,2590.65,  IX401550
X2700.65/                                                          IX401560
 DATA (LMB(I),I=1,14)/      3.E-3,5.E-3,10.E-3,20.E-3,15.E-3,10.E-3,IX401570
X7.E-3,5.E-3,4.E-3,3.3E-3,2.6E-3,1.7E-3,1.1E-3,1.1E-3/             IX401580
 DATA (MB(I),I=1,14)/  28.9644,28.88,28.56,28.07,26.92,26.66,26.40,IX401590
X25.85,24.70,22.66,19.94,17.94,16.84,16.17/                        IX401600
 DATA (DMB(I),I=1,14)/      -0.844E-5,-3.20E-5,-4.9E-5,-3.833E-5,  IX401610
X2*-2.60E-5,-2.75E-5,-2.875E-5,-2.914E-5,-2.72E-5,-2.0E-5,-1.1E-5, IX401620
X-0.67E-5,-0.67E-5/                                                IX401630
 DATA (PB(I),I=1,14)/.172244361E-4,.315971712E-5,.774389807E-6,    IX401640
X.265977111E-6,.535849383E-7,.391284945E-7,.295911117E-7,          IX401650
X.178715656E-7,.739258171E-8,.200573116E-8,.430456606E-9,          IX401660
X.117315480E-9,.370198961E-10,.128115330E-10/                      IX401670
  END                                                              IX401680
```

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A CALCULATION METHOD FOR THE ABLATION OF GLASS-TIPPED BLUNT BODIES

By John D. Warmbrod

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Werner K. Dahm
Chief, Aerodynamics Division

E. D. Geissler
Director, Aero-Astrodynamics Laboratory

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